WBJEE - 2022

Answer Keys by

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MATHEMATICS

Q.No.	A	B	C	D
01	D	C	D	A
02	B	A	C	B
03	B	B	C	B
04	C	D	A	A
05	B	B	C	C
06	B	D	A	B
07	B	D	A	A
08	D	A	A	C
09	D	C	B	D
10	B	D	A	C
11	A	A	B	B
12	B	C	B	A
13	C	C	A	B
14	A	A	C	B
15	D	C	D	B
16	D	A	A	B
17	B	A	C	D
18	C	B	B	A
19	D	B	C	D
20	A	A	B	B
21	D	C	A	D
22	C	B	B	C
23	A	A	B	B
24	C	C	D	D
25	C	D	B	B
26	A	A	D	C
27	A	A	A	B
28	A	B	C	B
29	A	C	B	B
30	B	B	D	D
31	B	B	D	D
32	A	B	B	B
33	C	A	B	A
34	B	D	B	B
35	A	B	C	C
36	C	D	D	A
37	D	D	B	D
38	B	C	B	D
39	A	B	A	B
40	B	B	D	C
41	C	D	B	D
42	B	C	A	A
43	B	B	B	D
44	D	B	C	C
45	A	B	B	A
46	D	D	D	C
47	B	B	D	A
48	B	B	A	C
49	D	A	C	A
50	C	D	D	A
51	A	D	D	D
52	C	C	B	C
53	C	B	B	B
54	D	D	D	A
55	B	B	D	D
56	B	A	C	B
57	D	D	B	D
58	B	D	A	B
59	A	C	B	C
60	D	B	B	A
61	D	D	D	D
62	D	B	A	C
63	B	B	C	B
64	D	C	C	B
65	B	A	D	D
66	C	A, B,C	C	B, D
67	B, C	C	A, C, D	A, D
68	B	C	A, D	C
69	C	A, C, D	B, D	B, C
70	A, B ,C	C	B	B
71	C	B, D	C	C
72	C	A, D	B, C	A, B, C
73	A, C, D	B, C	C	C
74	A. D	B	C	A, C, D
75	B, D	C	A, B ,C	C

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Code - B

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ANSWERS & HINTS
for
WBJEE - 2022
SUB : MATHEMATICS

CATEGORY - I (Q:1 to Q50)

(Carry 1 mark each. Only one option is correct. Negative marks – ¼)

1. Two circle $S_1 = px^2 + py^2 + 2g'x + 2f'y + d = 0$ and $S_2 = x^2 + y^2 + 2gx + 2fy + d' = 0$ have a common chord PQ. The equation of PQ is

(A) $S_1 - S_2 = 0$ (B) $S_1 + S_2 = 0$ (C) $S_1 - pS_2 = 0$ (D) $S_1 + pS_2 = 0$

Ans : (C)

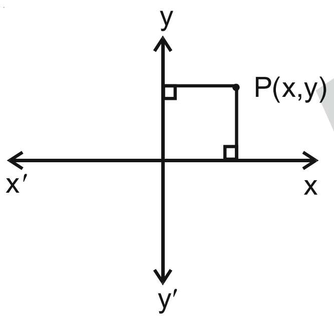
Hint : $\frac{S_1}{p} - S_2 = 0 \Rightarrow S_1 - pS_2 = 0$

2. If the sum of the distances of a point from two perpendicular lines in a plane is 1 unit, then its locus is

(A) a square (B) a circle (C) a straight line (D) two intersecting lines

Ans : (A)

Hint :



$|x| + |y| = 1$

Locus is square

3. A line passes through the point (–1, 1) and makes an angle $\sin^{-1}\left(\frac{3}{5}\right)$ in the positive direction of x-axis. If this line meets the curve $x^2 = 4y - 9$ at A and B, then |AB| is equal to

(A) $\frac{4}{5}$ unit (B) $\frac{5}{4}$ unit (C) $\frac{3}{5}$ unit (D) $\frac{5}{3}$ unit

Ans : (B)

Hint : $4y - 3x = 7$ and $x^2 = 4y - 9$

On solving, $x^2 - 3x + 2 = 0 \Rightarrow x = 1, 2$ and $y = \frac{5}{2}, \frac{13}{4}$

$A\left(1, \frac{5}{2}\right)$ and $B\left(2, \frac{13}{4}\right)$

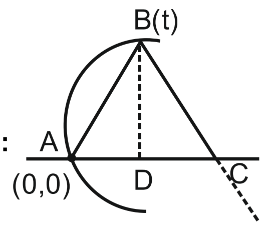
$|AB| = \frac{5}{4}$

4. AB is a chord of a parabola $y^2 = 4ax$, $(a > 0)$ with vertex A. BC is drawn perpendicular to AB meeting the axis at C. The projection of BC on the axis of the parabola is

(A) a unit (B) 2 a unit (C) 8a unit (D) 4a unit

Ans : (D)

Hint :



$B(at^2, 2at) \therefore D(at^2, 0)$

$m_{AB} = \frac{2at}{at^2} = \frac{2}{t} \therefore m_{BC} = -\frac{t}{2}$

$\therefore y - 2at = -\frac{t}{2}(x - at^2)$ [Equation of BC]

for $y = 0$, $-2at = -\frac{t}{2}(x - at^2) = -\frac{t}{2}x + \frac{at^3}{2}$

$\Rightarrow tx = 4at + at^3$

$\Rightarrow x = 4a + at^2$

$\therefore c(4a + at^2, 0)$

$\therefore$ Projection of BC on the axis is $DC = AC - AD = 4a + at^2 - at^2 = 4a$

5. Let $P(3 \sec\theta, 2\tan\theta)$ $Q(3 \sec\phi, 2\tan\phi)$ be two points on $\frac{x^2}{9} - \frac{y^2}{4} = 1$ such that $\theta + \phi = \frac{\pi}{2}$. $0 < \theta, \phi < \frac{\pi}{2}$. Then the ordinate of the point of intersection of the normals at P and Q is

(A) $\frac{13}{2}$ (B) $-\frac{13}{2}$ (C) $\frac{5}{2}$ (D) $-\frac{5}{2}$

Ans : (B)

Hint : $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$, $P(3\sec\theta, 2\tan\theta)$

$\therefore \frac{9x}{3\sec\theta} + \frac{4y}{2\tan\theta} = 9 + 4 = 13$

$\Rightarrow 3x\cos\theta + 2y\cot\theta = 13$ ...... (1)

Similarly, $3x\cos\phi + 2y\cot\phi = 13$

$$3x\cos\left(\frac{\pi}{2}-\theta\right)+2y\cot\left(\frac{\pi}{2}-\theta\right)=13$$

$\Rightarrow 3x\sin\theta + 2y\tan\theta = 13$ ...... (2)

$$\therefore \frac{13-2y\cot\theta}{3\cos\theta}=\frac{13-2y\tan\theta}{3\sin\theta}$$

$\Rightarrow 13\sin\theta - 2y\cos\theta = 13\cos\theta - 2y\sin\theta$

$\Rightarrow 13\tan\theta - 2y = 13 - 2y\tan\theta$

$\Rightarrow 13(\tan\theta - 1) = -2y(\tan\theta - 1)$

$$\therefore y = -\frac{13}{2}$$

$$\therefore \text{Ordinate} = -\frac{13}{2}$$

6. Let P be a point on (2, 0) and Q be a variable point on $(y-6)^2 = 2(x-4)$. Then the locus of mid-point of PQ is

(A) $y^2 + x + 6y + 12 = 0$ (B) $y^2 - x + 6y + 12 = 0$ (C) $y^2 + x - 6y + 12 = 0$ (D) $y^2 - x - 6y + 12 = 0$

Ans : (D)

Hint : P(2, 0)

$$Q\left(4+\frac{1}{2}t^2, 6+t\right)$$

Let midpoint of PQ be R(h, k)

$$h = 3+\frac{1}{4}t^2,\ k = 3+\frac{t}{2}$$

$\Rightarrow (k-3)^2 = (h-3)$

$\Rightarrow y^2 - 6y - x + 12 = 0$

7. The line $x - 2y + 4z + 4 = 0$, $x + y + z - 8 = 0$ intersect the plane $x - y + 2z + 1 = 0$ at the point

(A) (–2, 5, 1) (B) (2, –5, 1) (C) (2, 5, –1) (D) (2, 5, 1)

Ans : (D)

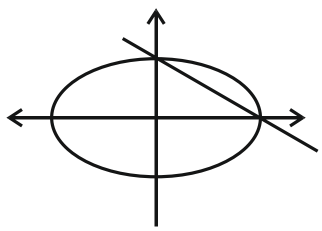
Hint : All other options are discarded

8. AB is a variable chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. If AB subtends a right angle at the origin O, then $\frac{1}{OA^2}+\frac{1}{OB^2}$ equals to

(A) $\frac{1}{a^2}+\frac{1}{b^2}$ (B) $\frac{1}{a^2}-\frac{1}{b^2}$ (C) $a^2 + b^2$ (D) $a^2 - b^2$

Ans : (A)

Hint :



9. The equation of the plane through the intersection of the planes x + y + z = 1 and 2x + 3y – z + 4 and parallel to the x-axis is

(A) $y + 3z + 6 = 0$ (B) $y + 3z - 6 = 0$ (C) $y - 3z + 6 = 0$ (D) $y - 3z - 6 = 0$

Ans : (C)

Hint : $(2x + 3y - z + 4) + \lambda(x \ \ y + z - 1) = 0$

$\vec{n} = (2+\lambda)\hat{i} + (3+\lambda)\hat{j} + (-1+\lambda)\hat{k}$

$\vec{n}.\vec{i} = 0 \Rightarrow \lambda = -2$

$\Rightarrow y - 3z + 6 = 0$

10. The values of a, b, c for which the function $f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ c, & x = 0 \\ \dfrac{(x+bx^2)^{1/2} - \sqrt{x}}{bx^{1/2}}, & x > 0 \end{cases}$

is continuous at x = 0, are

(A) $a = \frac{3}{2}, b = -\frac{3}{2}, c = \frac{1}{2}$

(B) $a = -\frac{3}{2}, c = \frac{3}{2}$, b is arbitrary non-zero real number

(C) $a = -\frac{5}{2}, b = -\frac{3}{2}, c = \frac{3}{2}$

(D) $a = -2, b \in \mathbb{R} - \{0\}, c = 0$

Ans : (D)

Hint : $f(0^+) = \underset{x \to 0^+}{Lt} \dfrac{(1+bx)^{1/2} - 1}{b} = 0$

$f(0) = c$

$f(0^-) = a + 2$

$\Rightarrow c = 0$ and $a = -2,\ b \in R - \{0\}$

11. If $y = e^{\tan^{-1}x}$ then

(A) $(1 + x^2)y_2 + (2x - 1)y_1 = 0$

(B) $(1 + x^2)y_2 + 2xy = 0$

(C) $(1 + x^2)y_2 - y_1 = 0$

(D) $(1 + x^2)y_2 + 3xy_1 + 4y = 0$

Ans : (A)

Hint : $\dfrac{dy}{dx} = e^{\tan^{-1}x} \times \dfrac{1}{1+x^2}$

$\Rightarrow (1 + x^2)y' = y$

or $2xy' + (1 + x^2)y'' = y'$

or $(1 + x^2)y'' + y'(2x - 1) = 0$

12. Domain of $y=\sqrt{\log_{10}\frac{3x-x^2}{2}}$ is

(A) $x < 1$ (B) $2 < x$ (C) $1 \le x \le 2$ (D) $2 < x < 3$

Ans : (C)

Hint : $\log_{10}\left(\frac{3x-x^2}{2}\right) \ge 0$

or $\log_{10}\left(\frac{3x-x^2}{2}\right) \ge \log_{10} 1$

or $x^2 - 3x + 2 \le 0$

or $x \in [1, 2]$ and $\frac{3x-x^2}{2} > 0$

$\Rightarrow x \in [1, 2]$

13. Let $f(x) = a_0 + a_1|x| + a_2|x|^2 + a_3|x|^3$, where a_0, a_1, a_2, a_3 are real constants. Then f(x) is differentiable at x = 0

(A) whatever be a_0, a_1, a_2, a_3

(B) for no values of a_0, a_1, a_2, a_3

(C) only if $a_1 = 0$

(D) only if $a_1 = 0, a_3 = 0$

Ans : (C)

Hint : $f'(0^+) = \lim_{x\to 0}(a_1 + 2a_1x + 3a_3x^2) = a_1$

$f'(0^-) = \lim_{x\to 0}(-a_1 + 2a_1x - 3a_3x^2) = -a_1$

$f'(0^+) = f'(0^-)$ $\Rightarrow a_1 = 0$

14. Let $f:[a, b] \to \mathbb{R}$ be continuous in [a, b], differentiable in (a, b) and f(a) = 0 = f(b). Then

(A) there exists at least one point $c \in (a, b)$ for which $f'(c) = f(c)$

(B) $f'(x) = f(x)$ does not hold at any point of (a, b)

(C) at every point of (a, b), $f'(x) > f(x)$

(D) at every point of (a, b), $f'(x) < f(x)$

Ans : (A)

Hint : Let $g(x) = e^{-x}f(x)$

$g(a) = g(b) = 0$

By Rolle's theorem, for atleast one $c \in (a, b)$ where

$g'(c) = 0$

or $e^{-c}f'(c) - f(c)e^{-c} = 0$

or $e^{-c}(f'(c) - f(c)) = 0$

or $f'(c) = f(c)$ for atleast one $c \in (a, b)$

15. $\lim_{x\to 0}\left(\frac{1}{x}\ln\sqrt{\frac{1+x}{1-x}}\right)$ is

(A) ½ (B) 0 (C) 1 (D) does not exist

Ans : (C)

Hint : $\lim_{x\to 0}\frac{\frac{1}{2}\left(\ln(1+x)-\ln(1-x)\right)}{x}$

$= \frac{1}{2}\lim_{x\to 0}\left(\frac{1}{1+x}+\frac{1}{1-x}\right)=1$

16. Let f be derivable in [0, 1], then

(A) there exists $c\in(0, 1)$ such that $\int_0^c f(x)dx = (1-c)f(c)$

(B) there does not exist any point $d\in(0, 1)$ for which $\int_0^d f(x)dx = (1-d)f(d)$

(C) $\int_0^c f(x)dx$ does not exist, for any $c\in(0, 1)$

(D) $\int_0^c f(x)dx$ is independent of c, $c\in(0, 1)$

Ans : (A)

Hint : Let g(x) = $x\int_0^x f(t)dt - \int_0^x f(t)dt$

Now, g(0) = 0

g(1) = 0

By Rolle's Theorem

$g'(x) = 0$ for some $x\in(0, 1)$

$g'(x) = xf(x) + \int_0^x f(t)dt - f(x) = 0$

17. $I = \int\cos(\ln x)dx$. Then I =

(A) $\frac{x}{2}\{\cos(\ln x)+\sin(\ln x)\}+c$

(B) $x^2\{\cos(\ln x) - \sin(\ln x)\} + c$

(C) $x^2 \sin(\ln x) + c$

(D) $x\cos(\ln x) + c$

Ans : (A)

Hint : $\int\cos(\ln(x))dx = x\cdot\cos(\ln x) - \int\left(-\sin(\ln x)\right)\times \frac{1}{x}(x)dx$

$= x\cos(\ln x) + \int\sin(\ln x)dx$

$= x\cos(\ln x) + x\sin(\ln x) - \int\cos(\ln x)dx$

18. Let $\lim_{\epsilon\to 0+}\int_{\epsilon}^{x}\frac{bt\cos 4t - a\sin 4t}{t^2}dt = \frac{a\sin 4x}{x}-1, \left(0<x<\frac{\pi}{4}\right)$. Then a and b are given by

(A) a = 2, b = 2

(B) $a = \frac{1}{4}$, b = 1

(C) a = –1, b = 4

(D) a = 2, b = 4

Ans : (B)

Hint : Let $g(x)=\lim_{\epsilon\to 0}\int_{\epsilon}^{x}\frac{bt\cos 4t-a\sin 4t}{t^2}dt=\frac{a\sin 4x}{x}-1$

$\lim_{x\to 0}g(x)=0$ = 4a – 1 $\Rightarrow a=\frac{1}{4}$

$g'(x)=\frac{bx\cos 4x-a\sin 4x}{x^2}$

Comparing, b = 4a = 1

19. Let $\int\frac{x^{1/2}}{\sqrt{1-x^3}}dx=\frac{2}{3}g(f(x))+c;$ then

(A) $f(x)=\sqrt{x}, g(x)=x^{3/2}$

(B) $f(x)=x^{3/2}, g(x)=\sin^{-1}x$

(C) $f(x)=\sqrt{x}, g(x)=\sin^{-1}x$

(D) $f(x)=\sin^{-1}x, g(x)=x^{3/2}$

Ans : (B)

Hint : $\int\frac{x^{1/2}dx}{\sqrt{1-\left(x^{3/2}\right)^2}}$ = $\frac{2}{3}\int\frac{dt}{\sqrt{1-t^2}}$

$x^{3/2}$ = t

$\frac{3}{2}x^{1/2}dx=dt$

= $\frac{2}{3}\sin^{-1}(t)+c$

= $\frac{2}{3}\sin^{-1}\left(x^{3/2}\right)+c$

20. The value of $\int_0^{\pi/2}\frac{(\cos x)^{\sin x}}{(\cos x)^{\sin x}+(\sin x)^{\cos x}}dx$ is

(A) $\pi/4$

(B) 0

(C) $\pi/2$

(D) $1/2$

Ans : (A)

Hint : $I=\int_0^{\pi/2}\frac{(\cos x)^{\sin x}}{(\cos x)^{\sin x}+(\sin x)^{\cos x}}dx$

$I=\int_0^{\pi/2}\frac{(\sin x)^{\cos x}dx}{(\sin x)^{\cos x}+(\cos x)^{\sin x}}$

$\Rightarrow 2I=\pi/2$

$\Rightarrow I=\pi/4$

21. A curve passes through the point (3, 2) for which the segment of the tangent line contained between the co-ordinate axes is bisected at the point of contact. The equation of the curve is

(A) $y = x^2 - 7$ (B) $x = \frac{y^2}{2} + 2$

(C) $xy = 6$ (D) $x^2 + y^2 - 5x + 7y + 11 = 0$

Ans : (C)

Hint : $\frac{x}{x_1} + \frac{y}{y_1} = 2$

$\because \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = -\frac{y_1}{x_1}$

$\therefore \frac{dy}{dx} = -\frac{y}{x} \Rightarrow \frac{dy}{y} = -\frac{dx}{x} \Rightarrow \ln y = -\ln x + k \Rightarrow \ln xy = k \Rightarrow xy = e^k = C \Rightarrow xy = C$ passes through (3, 2)

$\therefore C = 3 \times 2 = 6 \therefore xy = 6$

22. Let $f(x) = \int_{\sin x}^{\cos x} e^{-t^2} dt$. Then $f'\left(\frac{\pi}{4}\right)$ equals

(A) $\sqrt{1/e}$ (B) $-\sqrt{2/e}$ (C) $\sqrt{2/e}$ (D) $-\sqrt{1/e}$

Ans : (B)

Hint : $f'(x) = -e^{-\cos^2 x} \sin x - e^{-\sin^2 x} \cos x$

$\therefore f'\left(\frac{\pi}{4}\right) = -e^{-\frac{1}{2}} \frac{1}{\sqrt{2}} - e^{-\frac{1}{2}} \frac{1}{\sqrt{2}} = -\frac{2}{\sqrt{2e}} = -\sqrt{\frac{2}{e}}$

23. If $x\frac{dy}{dx} + y = x\frac{f(xy)}{f'(xy)}$, then |f(xy)| is equal to

(A) $Ce^{x^2/2}$ (B) Ce^{x^2} (C) Ce^{2x^2} (D) $Ce^{x^2/3}$

where C is the constant of integration

Ans : (A)

Hint : $x\frac{dy}{dx} + y = x\frac{f(xy)}{f'(xy)} \Rightarrow \frac{d(xy)}{dx} = x\frac{f(xy)}{f'(xy)} \Rightarrow \frac{f'(xy)}{f(xy)} d(xy) = xdx \Rightarrow \ln|f(xy)| = \frac{x^2}{2} + k \Rightarrow f(xy) = Ce^{\frac{x^2}{2}}$

24. Let $f(x) = (x-2)^{17}(x+5)^{24}$. Then

(A) f does not have a critical point at x = 2 (B) f has a minimum at x = 2

(C) f has neither a maximum nor a minimum at x = 2 (D) f has a maximum at x = 2

Ans : (C)

Hint : $f(x) = (x-2)^{17}(x+5)^{24}$

$\Rightarrow f'(x) = 17(x-2)^{16}(x+5)^{24} + 24(x-2)^{17}(x+5)^{23} = (x-2)^{16}(x+5)^{23}(17x+85+24x-48) = (x-2)^{16}(x+5)^{23}(41x+37)$

+ – + +

$-5 \quad \frac{-37}{41} \quad 2$

No sign change at x = 2
$\Rightarrow$f(x) has neither a maximum nor a minimum at x = 2

25. The solution of $\cos y \frac{dy}{dx} = e^{x+\sin y} + x^2 e^{\sin y}$ is f(x) + $e^{-\sin y}$ = C (C is arbitrary real constant) where f(x) is equal to

(A) $e^x + \frac{1}{2}x^3$ (B) $e^{-x} + \frac{1}{3}x^3$ (C) $e^{-x} + \frac{1}{2}x^3$ (D) $e^x + \frac{1}{3}x^3$

Ans : (D)

Hint : $-e^{-\sin y} \cos y \frac{dy}{dx} = -[e^x + x^2] \Rightarrow d(e^{-\sin y}) + (e^x + x^2)dx = 0 \Rightarrow e^{-\sin y} + e^x + \frac{x^3}{3} = C$

26. The point of contact of the tangent to the parabola $y^2 = 9x$ which passes through the point (4, 10) and makes an angle θ with the positive side of the axis of the parabola where $\tan\theta > 2$, is

(A) $\left(\frac{4}{9}, 2\right)$ (B) (4, 6) (C) (4, 5) (D) $\left(\frac{1}{4}, \frac{1}{6}\right)$

Ans : (A)

Hint : $\tan\theta = \frac{1}{t} > 2$

Equation of tangent at t

$yt = x + \frac{9}{4}t^2$ (passes through (4,10))$\Rightarrow 10t = 4 + \frac{9}{4}t^2 \Rightarrow 9t^2 - 40t + 16 = 0 \Rightarrow t = 4, \frac{4}{9} \left(\frac{1}{t} > 2\right) \Rightarrow t = \frac{4}{9}$

$\therefore$ point $(at^2, 2at) \Rightarrow \left(\frac{4}{9}, 2\right)$

27. A particle moving in a straight line starts from rest and the acceleration at any time t is $a - kt^2$ where a and k are positive constants. The maximum velocity attained by the particle is

(A) $\frac{2}{3}\sqrt{a^3/k}$ (B) $\frac{1}{3}\sqrt{a^3/k}$ (C) $\sqrt{a^3/k}$ (D) $2\sqrt{a^3/k}$

Ans : (A)

Hint : $\frac{dv}{dt} = a - kt^2 = \left(\sqrt{a} - \sqrt{k}t\right)\left(\sqrt{a} + \sqrt{k}t\right) \; [\because a, k > 0]$

– + –

$-\sqrt{\frac{a}{k}} \qquad \sqrt{\frac{a}{k}}$

v will be max at $t = \sqrt{\frac{a}{k}}$

$\because v = at - \frac{k}{3}t^3 + C$

at t = 0, v = 0 $\Rightarrow$ C = 0

$\therefore v = at - \frac{k}{3}t^3$

$\therefore v_{max} = a\sqrt{\frac{a}{k}} - \frac{k}{3}\frac{a}{k}\sqrt{\frac{a}{k}}$

$= \frac{2a}{3}\sqrt{\frac{a}{k}}$

28. If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ is unit vector perpendicular to $\vec{a}$ and coplanar with $\vec{a}$ and $\vec{b}$, then unit vector $\vec{d}$ perpendicular to both $\vec{a}$ and $\vec{c}$ is

(A) $\pm\frac{1}{\sqrt{6}}\left(2\hat{i} - \hat{j} + \hat{k}\right)$ (B) $\pm\frac{1}{\sqrt{2}}\left(\hat{j} + \hat{k}\right)$ (C) $\pm\frac{1}{\sqrt{6}}\left(\hat{i} - 2\hat{j} + \hat{k}\right)$ (D) $\pm\frac{1}{\sqrt{2}}\left(\hat{j} - \hat{k}\right)$

Ans : (B)

Hint : $\because \vec{d}$ is normal to the plane of $\vec{a}$ and $\vec{b}$

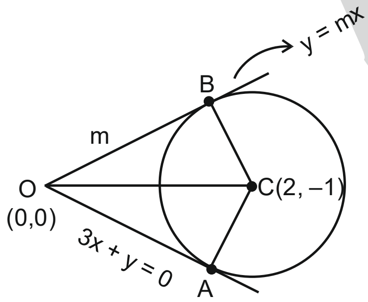
$\vec{a} \times \vec{b}$ or $\vec{b} \times \vec{a} = \pm 2\left(\hat{j} + \hat{k}\right)$ $\therefore$ unit vector $\vec{d} = \pm\frac{1}{\sqrt{2}}\left(\hat{j} + \hat{k}\right)$

29. If the equation of one tangent to the circle with centre at (2, –1) from the origin is 3x + y = 0, then the equation of the other tangent through the origin is

(A) 3x – y = 0 (B) x + 3y = 0 (C) x – 3y = 0 (D) x + 2y = 0

Ans : (C)

Hint :



$\because$ CA = CB

$\Rightarrow \frac{5}{\sqrt{10}} = \left|\frac{2m+1}{\sqrt{1+m^2}}\right|$

squaring

$\frac{\cancel{25}^{\,5}}{\cancel{10}_{\,2}} = \frac{4m^2 + 4m + 1}{1 + m^2}$

$\Rightarrow 5 + 5m^2 = 8m^2 + 8m + 2 \Rightarrow 3m^2 + 8m - 3 = 0 \Rightarrow m = \frac{1}{3}, -3$

$\therefore$ Equation of tangent OB is $y = \frac{x}{3}$

30. Area of the figure bounded by the parabola $y^2 + 8x = 16$ and $y^2 - 24x = 48$ is

(A) $\frac{11}{9}$ sq. unit (B) $\frac{32}{3}\sqrt{6}$ sq. unit (C) $\frac{16}{3}$ sq. unit (D) $\frac{24}{5}$ sq. unit

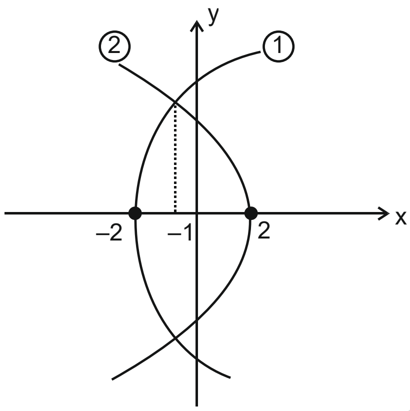
Ans : (B)

Hint : $y^2 = -8(x-2)$ --------(1)

$y^2 = 24(x+2)$ -----(2)

Solving (1) and (2) $\Rightarrow x = -1$

$$\text{Area} = 2\left[\int_{-2}^{-1} 2\sqrt{6}\sqrt{x+2}\,dx + \int_{-1}^{2} 2\sqrt{2}\sqrt{2-x}\,dx\right] = \frac{32}{3}\sqrt{6} \text{ sq. unit}$$



31. Let $a_n = (1^2 + 2^2 + ...n^2)^n$ and $b_n = n^n (n!)$. Then

(A) $a_n < b_n \ \forall n$ (B) $a_n > b_n \ \forall n$

(C) $a_n = b_n$ for infinitely many n (D) $a_n < b_n$ if n be even and $a_n > b_n$ if n be odd

Ans : (B)

Hint : Using PMI $a_n > b_n$ [* $a_n > b_n \ \forall n \geq 2$ as $a_1 = b_1$]

32. If a, b, c are in G. P. and log a. – log 2b, log 2b – log 3c, log 3c – log a are in A. P., then a, b, c are the lengths of the sides of a triangle which is

(A) acute angled (B) obtuse angled (C) right angled (D) equilateral

Ans : (B)

Hint : $\because 2\log\frac{2b}{3c} = \log\frac{a}{2b} + \log\frac{3c}{a} \Rightarrow \frac{4b^2}{9c^2} = \frac{3c}{2b} \Rightarrow 2b = 3c \Rightarrow \frac{c}{b} = \frac{2}{3}$ (common ratio)

$\therefore$ Sides are a, $\frac{2a}{3}, \frac{4a}{9}$

Using Cosine rule

$\Rightarrow \cos A < 0 \Rightarrow$ obteuse angled tiangle

33. If a, b are odd integers, then the roots of the equation $2ax^2 + (2a + b)x + b = 0$, $a \neq 0$ are

(A) rational (B) irrational (C) non-real (D) equal

Ans : (A)

Hint : Dise is a perfect square $\Rightarrow$ Rational roots

34. The number of zeros at the end of $\underline{|100}$ is

(A) 21 (B) 22 (C) 23 (D) 24

Ans : (D)

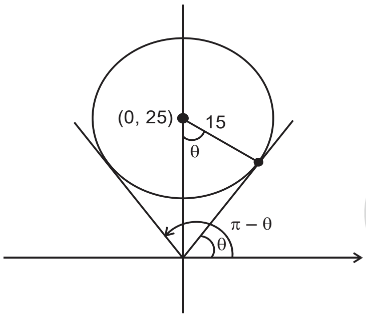
Hint : $E_5(100!) = \left[\frac{100}{5}\right] + \left[\frac{100}{25}\right] + \left[\frac{100}{125}\right]$ = 20 + 4 + 0 = 24

35. If $|z - 25i| \le 15$, then Maximum arg(z) – Minimum arg(z) is equal to

(A) $2\cos^{-1}\left(\frac{3}{5}\right)$ (B) $2\cos^{-1}\left(\frac{4}{5}\right)$ (C) $\frac{\pi}{2} + \cos^{-1}\left(\frac{3}{5}\right)$ (D) $\sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{3}{5}\right)$

Ans : (B)

Hint :



$\because \cos\theta = \frac{15}{25} = \frac{3}{5}$

$\therefore$ Min arg (z) = $\cos^{-1}\left(\frac{3}{5}\right)$

Max arg (z) = $\pi - \cos^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2} + \sin^{-1}\left(\frac{3}{5}\right)$

$\therefore$ difference = $\frac{\pi}{2} + \sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{3}{5}\right) = 2\sin^{-1}\left(\frac{3}{5}\right) = 2\cos^{-1}\left(\frac{4}{5}\right)$

36. If z = x – iy and $z^{1/3}$ = p + iq (x, y, p, q $\in \mathbb{R}$), then $\frac{\left(\frac{x}{p} + \frac{y}{q}\right)}{\left(p^2 + q^2\right)}$ is equal to

(A) 2 (B) –1 (C) 1 (D) –2

Ans : (D)

Hint : $z = p^3 - 3pq^2 + i(3p^2q - q^3) = x - iy$

$x = p(p^2 - 3q^2)$

$y = -q(3p^2 - q^2)$

$\therefore \frac{x}{p} + \frac{y}{q} = p^2 - 3q^2 - 3p^2 + q^2 = -2(p^2 + q^2)$

37. A is a set containing n elements. P and Q are two subsets of A. Then the number of ways of choosing P and Q so that $P \cap Q = \varphi$ is

(A) $2^{2n}\,{}^{2n}C_n$ (B) 2^n (C) $3^n - 1$ (D) 3^n

Ans : (D)

Hint : $\sum_{r=0}^{n} {}^nC_r.2^{n-r} = (2+1)^n = 3^n$ (Assuming P contains r elements then Q can be formed with n–r elements in 2^{n-r} ways)

38. There are n white and n black balls marked 1, 2, 3, ... n. The number of ways in which we can arrange these balls in a row so that neighbouring balls are of different colours is

(A) $(n!)^2$ (B) $(2n)!$ (C) $2(n!)^2$ (D) $\frac{(2n)!}{(n!)^2}$

Ans : (C)

Hint : BW BW ..... = n! × n!

or

WB WB ..... $= 2(n!)^2$

39. Let $f(n) = 2^{n+1}$, $g(n) = 1 + (n + 1)2^n$ for all $n \in \mathbb{N}$. Then

(A) f(n) > g(n) (B) f(n) < g(n)

(C) f(n) and g(n) are not comparable (D) f(n) > g(n) if n be even and f(n) < g(n) if n be odd

Ans : (B)

Hint : $g(n) - f(n) = 1 + (n + 1)2^n - 2.2^n = 1+n.2^n - 2^n = 1+ (n-1)2^n > 0$

$\because\ n \ge 1 \Rightarrow (n-1) \ge 0$

40. If $\Delta(x) = \begin{vmatrix} x-2 & (x-1)^2 & x^3 \\ x-1 & x^2 & (x+1)^3 \\ x & (x+1)^2 & (x+2)^3 \end{vmatrix}$, then coefficient of x in $\Delta(x)$ is

(A) 2 (B) –2 (C) 3 (D) –4

Ans : (B)

Hint : co-eff of x in $\Delta(x) = \frac{\Delta'(0)}{1!} = -2$

41. Under which of the following condition(s) does(do) the system of equations $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 1 & 2 & (a-4) \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ a \end{pmatrix}$ possesses (possess) unique solution ?

(A) $\forall a \in R$ (B) a = 8

(C) for all integral values of a (D) $a \neq 8$

Ans : (D)

Hint : $\Delta \neq 0$

$$\Rightarrow \begin{vmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 1 & 2 & a-4 \end{vmatrix} \neq 0 \Rightarrow a \neq 8$$

42. Let S, T, U be three non-void sets and $f : S \to T$, $g : T \to U$ and composed mapping $g.f : S \to U$ be defined. Let g. f be injective mapping. Then

(A) f, g both are injective.
(B) neither f nor g is injective.
(C) f is obviously injective.
(D) g is obviously injective.

Ans : (C)

Hint : Let, $x_1, x_2 \in S$ and $f(x_1) = f(x_2)$

$\Rightarrow gof\,(x_1) = g\,(f(x_1)) = g\,(f(x_2)) = gof\,(x_2)$

$\Rightarrow x_1 = x_2$ (as gof is injective)

$\Rightarrow f(x)$ is obviously injective.

43. If $p = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of the 3 × 3 matrix A and det A = 4, then α is equal to

(A) 4
(B) 11
(C) 5
(D) 0

Ans : (B)

Hint : $|p| = 2\alpha - 6 = (\det A)^2 = 16$

$\Rightarrow \alpha = 11$

44. If $A = \begin{pmatrix} 1 & 1 \\ 0 & i \end{pmatrix}$ and $A^{2018} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then (a + d) equals

(A) 1 + i
(B) 0
(C) 2
(D) 2018

Ans : (B)

Hint : $A^2 = \begin{pmatrix} 1 & 1+i \\ 0 & -1 \end{pmatrix}, A^3 = \begin{pmatrix} 1 & i \\ 0 & -i \end{pmatrix}, A^4 = I$

$\therefore A^{2018} = A^{2016} \times A^2 = A^2 = \begin{pmatrix} 1 & 1+i \\ 0 & -1 \end{pmatrix}$

$\therefore$ a + d = 1 + (–1) = 0

45. A determinant is chosen at random from the set of all determinants of order 2 with elements 0 or 1 only. The probability that the determinant chosen is non-zero is

(A) $\frac{3}{16}$
(B) $\frac{3}{8}$
(C) $\frac{1}{4}$
(D) $\frac{5}{8}$

Ans : (B)

Hint : $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc = 0$

Case 1 : ad = bc = 0 $\to$ 3 × 3 = 9 cases

Case 2 : ad = bc = 1 $\to$ 1 case only

$\therefore$ Probability = $1-\frac{9+1}{2^4}=\frac{3}{8}$

46. For the mapping $f:\mathbb{R}-\{1\}\to\mathbb{R}-\{2\}$, given by $f(x)=\frac{2x}{x-1}$, which of the following is correct ?

(A) f is one-one but not onto
(B) f is onto but not one-one
(C) f is neither one-one nor onto
(D) f is both one-one and onto

Ans : (D)

Hint : $f(x)=\frac{2x}{x-1}\Rightarrow f'(x)=\frac{-2}{(x-1)^2}<0$

$\therefore f(x)$ is one-one

Let, $y_1\in\mathbb{R}-\{2\}$ and $f(x_1)=y_1$

Then, $y_1=\frac{2x_1}{x_1-1}\Rightarrow x_1=\frac{y_1}{y_1-2}\in\mathbb{R}-\{1\}$

$\therefore$ f(x) is onto .

47. A, B, C are mutually exclusive events such that $P(A)=\frac{3x+1}{3}, P(B)=\frac{1-x}{4}$ and $P(C)=\frac{1-2x}{2}$. Then the set of possible values of x are in

(A) [0, 1]
(B) $\left[\frac{1}{3},\frac{1}{2}\right]$
(C) $\left[\frac{1}{3},\frac{2}{3}\right]$
(D) $\left[\frac{1}{3},\frac{13}{3}\right]$

Ans : (B)

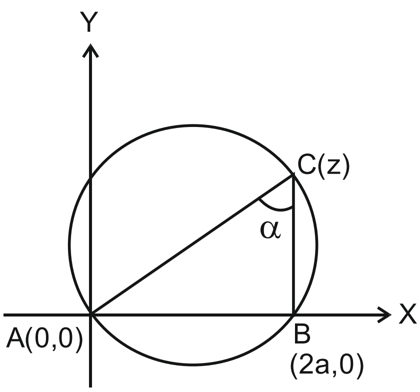
Hint : $0\le P(A)+P(B)+P(C)\le 1$ and $0\le P(A)\le 1,\ 0\le P(B)\le 1,\ 0\le P(C)\le 1\ \Rightarrow \frac{1}{3}\le x\le\frac{1}{2}$

48. The side AB of ΔABC is fixed and is of length 2a unit. The vertex moves in the plane such that the vertical angle is always constant and is α. Let x-axis be along AB and the origin be at A. Then the locus of the vertex is

(A) $x^2 + y^2 + 2ax \sin\alpha + a^2 \cos\alpha = 0$
(B) $x^2 + y^2 - 2ax - 2ay \cot\alpha = 0$
(C) $x^2 + y^2 - 2ax \cos\alpha - a^2 = 0$
(D) $x^2 + y^2 - ax \sin\alpha - ay \cos\alpha = 0$

Ans : (B)

Hint :



Let, c = z = x + iy

$\arg\left(\frac{2a-z}{0-z}\right)=\alpha$

$\Rightarrow \arg\left(\frac{(x-2a)+iy}{x+iy}\right)=\alpha$

$\Rightarrow x^2+y^2-2ax-2ay\cot\alpha=0$

49. If $(\cot \alpha_1)$ $(\cot \alpha_2)$ ...... $(\cot \alpha_n) = 1$, $0 < \alpha_1, \alpha_2, ... \alpha_n < \pi/2$, then the maximum value of $(\cos a_1)$ $(\cos \alpha_2)$ ...... $(\cos \alpha_n)$ is given by

(A) $\frac{1}{2^{n/2}}$ (B) $\frac{1}{2^n}$ (C) $\frac{1}{2n}$ (D) 1

Ans : (A)

Hint : $\frac{1}{(\cos\alpha_1 \cos\alpha_2 ......\cos\alpha_n)^2}$

$= (1+\tan^2\alpha_1)(1+\tan^2\alpha_2)......(1+\tan^2\alpha_n)$

$\geq (2\tan\alpha_1)(2\tan\alpha_2)......(2\tan\alpha_n)$ [AM – GM inequality]

$= 2^n \times \frac{1}{\cot\alpha_1 \cot\alpha_2 ......\cot\alpha_n} = 2^n$

$\therefore \cos\alpha_1 \cos\alpha_2 ......\cos\alpha_n \leq \frac{1}{2^{n/2}}$

50. If the algebraic sum of the distances from the points (2, 0), (0, 2) and (1, 1) to a variable straight line be zero, then the line passes through the fixed point

(A) (–1, 1) (B) (1, –1) (C) (–1, –1) (D) (1, 1)

Ans : (D)

Hint : $\left(\frac{2+0+1}{3}, \frac{0+2+1}{3}\right)$ i.e. (1,1).

CATEGORY - II (Q51 to Q65)

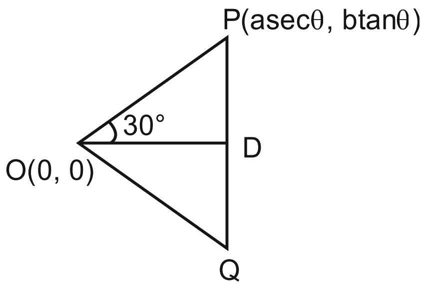
(Carry 2 marks each. Only one option is correct. Negative marks : ½)

51. PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that ΔOPQ is an equilateral triangle, O being the centre of the hyperbola. Then the eccentricity e of the hyperbola satisfies

(A) $1 < e < 2/\sqrt{3}$ (B) $e = 2/\sqrt{3}$ (C) $e = 2\sqrt{3}$ (D) $e > 2/\sqrt{3}$

Ans : (D)

Hint : $\tan 30^0 = \frac{b\tan\theta}{a\sec\theta}$



$\Rightarrow \frac{b}{a} = \frac{1}{\sin\theta\sqrt{3}}$

$e = \sqrt{1+\frac{1}{3\sin^2\theta}} > \sqrt{1+\frac{1}{3}}$

$\Rightarrow e > \frac{2}{\sqrt{3}}\left(0 < \sin^2\theta < 1\right)$

52. Let f be a non-negative function defined in [0, $\pi/2$], f′ exists and be continuous for all x and $\int_0^x \sqrt{1-\left(f'(t)\right)^2}\,dt = \int_0^x f(t)\,dt$ and f (0) = 0. Then

(A) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(B) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

(C) $f\left(\frac{4}{3}\right) < \frac{4}{3}$ and $f\left(\frac{2}{3}\right) < \frac{2}{3}$

(D) $f\left(\frac{4}{3}\right) > \frac{4}{3}$ and $f\left(\frac{2}{3}\right) > \frac{2}{3}$

Ans : (C)

Hint : $\int_0^x \sqrt{1-\left(f'(t)\right)^2}\,dt = \int_0^x f(t)\,dt$

Using Leibnitz Rule,

$\sqrt{1-\left(f'(x)\right)^2} = f(x)$

$\therefore f'(x) = \pm\sqrt{1-f^2(x)} \Rightarrow \int \frac{df(x)}{\sqrt{1-f^2(x)}} = \pm\int dx$

$\sin^{-1}(f(x)) = \pm x + c$

$\therefore f(x) = \sin(\pm x + C)$

$\because f(0) = 0 \Rightarrow C = 0$

$\therefore f(x) = \sin x$ or $-\sin x$

But f is non-negative on $\left[0, \pi/2\right]$

$\therefore f(x) = \text{Sin } x$

$\because f(x) < x$ for all $x > 0$

$\therefore f\left(\frac{4}{3}\right) < \frac{4}{3}$ and $f\left(\frac{2}{3}\right) < \frac{2}{3}$

53. If the transformation $z = \log \tan\frac{x}{2}$ reduces the differential equation $\frac{d^2y}{dx^2} + \cot x\frac{dy}{dx} + 4y\,\text{cosec}^2\,x = 0$ into the form $\frac{d^2y}{dz^2} + ky = 0$ then k is equal to

(A) – 4 (B) 4 (C) 2 (D) – 2

Ans : (B)

Hint : $\frac{d^2y}{dz^2} = \frac{d}{dz}\left(\frac{dy}{dz}\right) = \frac{d}{dz}\left(\frac{\frac{dy}{dx}}{\frac{dz}{dx}}\right)$

$$= \frac{d}{dz}\left(\frac{\frac{dy}{dx}}{\frac{1}{\sin x}}\right) \qquad \left(\because \frac{dz}{dx} = \frac{1}{\sin x}\right)$$

$$= \frac{d}{dz}\left(\sin x \cdot \frac{dy}{dx}\right) = \frac{\frac{d}{dx}\left(\sin x \cdot \frac{dy}{dx}\right)}{\frac{dz}{dx}}$$

$$= \sin x\left[\cos x \cdot \frac{dy}{dx} + \sin x \frac{d^2y}{dx^2}\right]$$

$$\because \frac{d^2y}{dz^2} + ky = 0$$

$$\therefore \sin x \cdot \cos x \cdot \frac{dy}{dx} + \sin^2 x \frac{d^2y}{dx^2} + ky = 0$$

Dividing the equation by $\sin^2 x$, we get $\frac{d^2y}{dx^2} + \cot x \cdot \frac{dy}{dx} + k \cdot \operatorname{cosec}^2 x \cdot y = 0$

Comparing with the equation

$$\frac{d^2y}{dx^2} + \cot x \cdot \frac{dy}{dx} + 4\operatorname{cosec}^2 x \cdot y = 0$$

We get k = 4

54. If I is the greatest of

$I_1 = \int_0^1 e^{-x} \cos^2 x \, dx,\ I_2 = \int_0^1 e^{-x^2} \cos^2 x \, dx,\ I_3 = \int_0^1 e^{-x^2} dx,\ I_4 = \int_0^1 e^{-x^2/2} dx$, then

(A) $I = I_1$ (B) $I = I_2$ (C) $I = I_3$ (D) $I = I_4$

Ans : (D)

Hint : $e^{-x} \cdot \cos^2 x < e^{-x^2} \cos^2 x < e^{-x^2} < e^{-\frac{x^2}{2}}$ for 0 < x < 1

$\therefore$ By domination Law, I_4 is maximum

$\therefore I = I_4$

55. $\lim_{x\to\infty}\left(\frac{x^2+1}{x+1} - ax - b\right)$, (a, b $\in$ R) = 0. Then

(A) a = 0, b = 1 (B) a = 1, b = –1 (C) a = –1, b – 1 (D) a = 0, b = 0

Ans : (B)

Hint : $\lim_{x\to\infty} \frac{x^2 + 1 - ax(x+1) - b(x+1)}{x+1} = 0$

$= \lim_{x\to\infty} \frac{(1-a)x^2 - (a+b)x + 1 - b}{x+1} = 0$

for the limit to exist,

$1 - a = 0$ and $-(a + b) = 0$

$\therefore$ a = 1, b = –1

56. Let the tangent and normal at any point $P(at^2, 2at)$, $(a > 0)$, on the parabola $y^2 = 4ax$ meet the axis of the parabola at T and G respectively. Then the radius of the circle through P, T and G is

(A) $a(1 + t^2)$ (B) $(1 + t^2)$ (C) $a(1 - t^2)$ (D) $(1 - t^2)$

Ans : (A)

Hint : $P(at^2, 2at)$, $T(-at^2, 0)$, $G(2a + at^2, 0)$

Slope of PT × Slope of PG = – 1

$\therefore$ TG is a diameter of the circle through points P, T and G.

$\therefore$ radius = $\frac{1}{2}$TG = $a(1 + t^2)$

57. From the point (–1, –6), two tangents are drawn to $y^2 = 4x$. Then the angle between the two tangents is

(A) $\pi/3$ (B) $\pi/4$ (C) $\pi/6$ (D) $\pi/2$

Ans : (D)

Hint : (–1, –6) lies on the directrix of the parabola $y^2 = 4x$.

$\therefore$ angle between tangents = $\pi/2$

58. If $\vec{\alpha}$ is a unit vector, $\vec{\beta} = \hat{i} + \hat{j} - \hat{k}$, $\vec{\gamma} = \hat{i} + \hat{k}$, then the maximum value of $[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}]$ is

(A) 3 (B) $\sqrt{3}$ (C) 2 (D) $\sqrt{6}$

Ans : (D)

Hint : $[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = \vec{\alpha}\cdot(\vec{\beta}\times\vec{\gamma}) = \vec{\alpha}\cdot\begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & 1 & -1 \\ 1 & 0 & 1 \end{vmatrix}$

$= \vec{\alpha}\cdot(\hat{i} + 2\hat{j} - \hat{k})$ is maximum $\Rightarrow$ angle between $\vec{\alpha}$ & $\hat{i} + 2\hat{j} - \hat{k}$ will be 0

$\therefore [\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}] = |\vec{\alpha}|\,|\hat{i} + 2\hat{j} - \hat{k}| = \sqrt{6}$

59. The maximum value of $f(x) = e^{\sin x} + e^{\cos x}$; $x \in R$ is

(A) 2e (B) $2\sqrt{e}$ (C) $2e^{1/\sqrt{2}}$ (D) $2e^{-1/\sqrt{2}}$

Ans : (C)

Hint : $f(x) = e^{\sin x} + e^{\cos x}$

$f'(x) = e^{\sin x}\cdot\cos x - e^{\cos x}\cdot\sin x$

$f''(x) = e^{\sin x}\cos^2 x + e^{\cos x}\cdot\sin^2 x - \sin x\cdot e^{\sin x} - \cos x\cdot e^{\cos x}$

$= e^{\sin x}(1 - \sin x - \sin^2 x) + e^{\cos x}(1 - \cos^2 x - \cos x)$

$f'(\pi/4) = 0$ and $f''(\pi/4) < 0$

$f'(x) = 0$ at $x = \pi/4 + 2n\pi$ or $5\pi/4 + 2n\pi$ $(n \in z)$

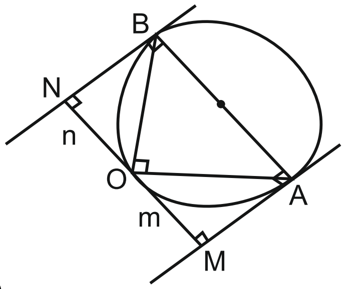
$f'(\pi/4 + 2n\pi) = 0$ and $f''(\pi/4 + 2n\pi) < 0$

$\therefore f_{max} = f(\pi/4 + 2n\pi), n \in z = 2e^{1/\sqrt{2}}$

60. A straight line meets the co-ordinate axes at A and B. A circle is circumscribed about the triangle OAB, O being the origin. If m and n are the distances of the tangent to the circle at the origin from the points A and B respectively, the diameter of the circle is

(A) m(m + n) (B) m + n (C) n(m + n) (D) $\frac{1}{2}(m+n)$

Ans : (B)



Hint : Clearly, AB is one of diameter

$\because$ AM and BN are parallel and $\angle BNO = \angle AMO = \pi/2$

$\therefore$ Points N, O and M are collinear.

$\therefore$ □BNMA is a rectangle

$\Rightarrow AB = MN = m + n$

61. The solution of $\det(A - \lambda I_2) = 0$ be 4 and 8 and $A = \begin{pmatrix} 2 & 3 \\ x & y \end{pmatrix}$. Then

(A) x = 4, y = 10 (B) x = 5, y = 8 (C) x = 3, y = 9 (D) x = –4, y = 10

Ans : (D)

Hint : $\det(A - \lambda I_2) = 0 \Rightarrow \begin{vmatrix} 2-\lambda & 3 \\ x & y-\lambda \end{vmatrix} = 0$

$\Rightarrow (2 - \lambda)(y - \lambda) - 3x = 0$

$\Rightarrow 2y - \lambda y - 2\lambda + \lambda^2 - 3x = 0$

$\Rightarrow \lambda^2 - (y + 2)\lambda + 2y - 3x = 0$

$y + 2 = 4 + 8 \Rightarrow y = 10$

and $2y - 3x = 4 \times 8 \Rightarrow x = -4$

62. The value of a for which sum of the squares of the roots of the equation $x^2 - (a-2)x - a - 1 = 0$ assumes the least value is

(A) 0 (B) 1 (C) 2 (D) 3

Ans : (B)

Hint : Let α, β be the roots, then

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (a - 2)^2 + 2(a + 1)$

$= a^2 - 2a + 6$

$= (a - 1)^2 + 5 \geq 5$

$\therefore \alpha^2 + \beta^2$ is minimum if a = 1.

63. If x satisfies the inequality $\log_{25}x^2 + (\log_5 x)^2 < 2$, then x belongs to

(A) $\left(\frac{1}{5}, 5\right)$ (B) $\left(\frac{1}{25}, 5\right)$ (C) $\left(\frac{1}{5}, 25\right)$ (D) $\left(\frac{1}{25}, 25\right)$

Ans : (B)

Hint : Domain = $(0, \infty)$

$$\log_5 x + \left(\log_5 x\right)^2 - 2 < 0$$

$$\Rightarrow \left(\log_5 x - 1\right)\left(\log_5 x + 2\right) < 0$$

$\therefore -2 < \log_5 x < 1$

$\therefore \frac{1}{25} < x < 5$

64. $f : X \to \mathbb{R}$, $X = \{x | 0 < x < 1\}$ is defined as $f(x) = \frac{2x-1}{1-|2x-1|}$. Then

(A) f is only injective
(B) f is only surjective
(C) f is bijective
(D) f is neither injective nor surjective

Ans : (C)

Hint : Put 2x –1 = t, then the function becomes

$$f(t) = \frac{t}{1-|t|}, -1 < t < 1$$

$$\therefore f(t) = \begin{cases} \frac{t}{1+t}, -1 < t \le 0 \\ \frac{t}{1-t}, 0 < t < 1 \end{cases}$$

$\because$ It is continuous and $f\left(-1^+\right) = -\infty$, $f\left(1^-\right) = +\infty$

$\therefore$ Range = $\left(-\infty, -\infty\right) = \mathbb{R}$

It is differentiable also,

$$f'(t) = \begin{cases} \frac{1}{(1+t)^2}, -1 < t < 0 \\ \frac{1}{(1-t)^2}, 0 < t < 1 \end{cases}$$

$\because \; f'(t) > 0 \forall -1 < t < 1$

$\therefore$ f is injective

65. If P_1P_2 and P_3P_4 are two focal chords of the parabola $y^2 = 4ax$ then the chords P_1P_3 and P_2P_4 intersect on the

(A) directrix of the parabola
(B) axis of the parabola
(C) latus-rectum of the parabola
(D) y-axis

Ans : (A)

Hint : Let $P_i(at_i^2, 2at_i)$, i = 1, 2, 3, 4

then, $t_1t_2 = t_3t_4 = -1$

equation of $P_1P_3 : (t_1 + t_3)y = 2x + 2at_1t_3$ .... (1)

$P_2P_4 : (t_2 + t_4)y = 2x + 2at_2t_4$ .... (2)

Putting x = –a in equation (1) gives $y=\frac{2at_1t_3-2a}{t_1+t_3}$

Putting x = –a in equation (2) gives

$$y=\frac{2at_2t_4-2a}{t_2+t_4}=\frac{\frac{2a}{t_1t_3}-2a}{-\frac{1}{t_1}-\frac{1}{t_3}}=\frac{2a(1-t_1t_3)}{-(t_1+t_3)}=\frac{2a(t_1t_3-1)}{(t_1+t_3)}$$

∴ These lines meet at $\left(-a,\frac{2a(t_1t_3-1)}{t_1+t_3}\right)$

CATEGORY - III (Q66 to Q75)

(Carry 2 marks each. One or more options are correct. No negative marks)

66. The line y = x + 5 touches

(A) the parabola $y^2 = 20x$

(B) the ellipse $9x^2 + 16y^2 = 144$

(C) the hyperbola $\frac{x^2}{29}-\frac{y^2}{4}=1$

(D) the circle $x^2 + y^2 = 25$

Ans : (A, B, C are correct)

Hint : (A) $y^2 = 20x = 4(5)x$ ∴ Tangent : $\boxed{y=mx+\frac{a}{m}}$ for m = 1, a = 5

∴ y = x + 5 is a tangent to $y^2 = 20x$

(B) $9x^2 + 16y^2 = 144$ $\boxed{c^2=a^2m^2+b^2}$ y = x + 5 ⇒ m = 1; c = 5

$\Rightarrow \frac{x^2}{16}+\frac{y^2}{9}=1$ ∴ $5^2 = 16(1)^2 + 9$ ⇒ True

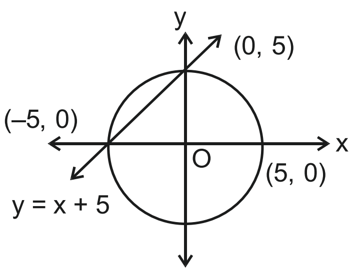
∴ y = x + 5 is a tangent to $9x^2 + 16y^2 = 144$

(C) $c^2 = a^2m^2 - b^2$ $\frac{x^2}{29}-\frac{y^2}{4}=1$ → $a^2 = 29$; $b^2 = 4$

$5^2 = 29(1)^2 - 4$ → True

∴ y = x + 5 is a tangent

(D)



∴ y = x + 5 is NOT tangent to $x^2 + y^2 = 25$

(A), (B), (C) are correct.

67. Let p(x) be a polynomial with real co-efficients, p(0) = 1 and $p'(x) > 0$ for all $x \in \mathbb{R}$. Then

(A) p(x) has at least two real roots

(B) p(x) has only one positive real root

(C) p(x) may have negative real root

(D) p(x) has infinitely many real roots

Ans : (C)

Hint : P(x) = 0 has exactly one negative real root.

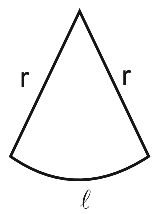
68. Twenty metres of wire is available to fence off a flower bed in the form of a circular sector. What must the radius of the circle be, if the area of the flower bed be greatest?

(A) 10 m (B) 4 m (C) 5 m (D) 6 m

Ans : (C)

Hint : Let, radius = r, arc length = ℓ

$2r + \ell = 20 \Rightarrow \ell = 20 - 2r$



$\Rightarrow A = \frac{1}{2}\ell r = \frac{1}{2}(20-2r)r \Rightarrow \frac{dA}{dr} = \frac{1}{2}(20-4r) = 0 \Rightarrow \boxed{r=5} \therefore r = 5m$

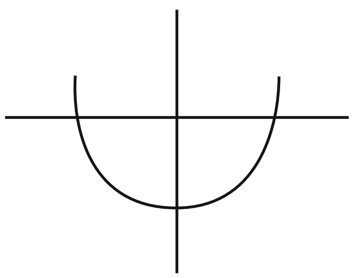
69. Let $f(x) = x^2 + x\sin x - \cos x$. Then

(A) f(x) = 0 has at least one real root (B) f(x) = 0 has no real root

(C) f(x) = 0 has at least one positive root (D) f(x) = 0 has at least one negative root

Ans : (A, C, D)

Hint : $f'(x) = 2x + x\cos x + \sin x + \sin x = 2(x + \sin x) + x\cos x$



$\Rightarrow f'(x) > 0 \ \forall x > 0, \ f'(x) < 0 \ \forall x < 0 \Rightarrow f(0) = -1$

70. From a balloon rising vertically with uniform velocity v ft/sec a piece of stone is let go. The height of the balloon above the ground when the stone reaches the ground after 4 sec is [$g = 32$ ft/sec^2]

(A) 220 ft (B) 240 ft (C) 256 ft (D) 260 ft

Ans : (C)

Hint : When stone is let go, its velocity = $-\nu$ (downwards)

Let, it was at a height h.

$\therefore h = -\nu t + \frac{1}{2}gt^2$