WBJEEM - 2015

MATHEMATICS

Q.No.	μ	β	γ	δ
01	B	A	A	D
02	B	A	C	A
03	B	C	A	*
04	C	B	B	C
05	D	D	B	A
06	A	A	B	C
07	A	*	C	A
08	D	C	D	A
09	C	C	A	*
10	C	B	D	D
11	B	C	A	A
12	D	A	A	B
13	A	C	A	B
14	*	A	C	B
15	C	A	B	C
16	B	D	A	B
17	B	A	D	B
18	D	*	B	D
19	D	D	D	D
20	B	A	C	A
21	A	B	C	B
22	B	C	C	A
23	C	A	B	B
24	A	B	D	A
25	A	B	C	D
26	B	B	C	A
27	A	C	D	D
28	A	B	A	B
29	C	D	*	A
30	A	C	C	A
31	D	B	A	A
32	A	B	*	C
33	A	D	D	B
34	A	C	A	C
35	C	C	B	B
36	B	D	B	C
37	B	C	D	A
38	D	D	B	D
39	C	A	A	B
40	D	C	B	C
41	C	C	A	A
42	C	B	D	B
43	A	B	B	D
44	*	A	A	C
45	D	B	C	C
46	A	A	B	C
47	B	C	C	D
48	A	B	B	C
49	B	A	C	C
50	A	D	A	C
51	C	A	C	B
52	B	A	C	C
53	C	B	D	B
54	C	A	C	A
55	B	D	A	D
56	A	B	B	C
57	C	A	A	C
58	C	D	C	A
59	D	C	B	B
60	C	C	A	A
61	C	D	A	C
62	D	A	A	A
63	B	C	C	A
64	D	C	D	A
65	A	A	B	D
66	C	C	C	A
67	A	D	A	C
68	A	B	D	C
69	C	A	A	D
70	A	A	C	B
71	C	A,B,C	A,B,D	B,C
72	A,C	B,C	A,C	B,C
73	A,B	B,C	C	A,B,D
74	A,D	B,C	A,C	A,C
75	A,B,D	A,B	B,C	A,B,C
76	A,C	A,D	B,C	B,C
77	A,B,C	A,B,D	A,B	C
78	B,C	A,C	A,D	A,C
79	B,C	C	A,B,C	A,B
80	B,C	A,C	B,C	A,D

* None of the options is correct

Code-μ

ANSWERS & HINT
for
WBJEEM - 2015
SUB : MATHEMATICS

CATEGORY - I (Q1 to Q60)

Each question has one correct option and carries 1 mark, for each wrong answer 1/4 mark will be deducted.

1. The value of $\lim\limits_{x\to 2}\int_2^x \frac{3t^2}{x-2}dt$ is

(A) 10 (B) 12
(C) 8 (D) 16

Ans : (B)

Hint : $\underset{x\to 2}{\text{Lt}} \frac{\frac{d}{dx}\left(\int_2^x 3t^2 dt\right)}{\frac{d}{dx}(x-2)} = 3\times 4 = 12$

2. If $\cot\frac{2x}{3} + \tan\frac{x}{3} = \operatorname{cosec}\frac{kx}{3}$, then the value of k is

(A) 1 (B) 2
(C) 3 (D) –1

Ans : (B)

Hint : $\cot 2\theta + \tan\theta = \operatorname{cosec} 2\theta \Rightarrow k = 2$

3. If $\theta \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$, then the value of $\sqrt{4\cos^4\theta + \sin^2 2\theta}$ + $4\cot\theta\cos^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$ is

(A) $-2\cot\theta$ (B) $2\cot\theta$
(C) $2\cos\theta$ (D) $2\sin\theta$

Ans : (B)

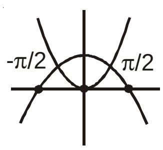
Hint : $|2\cos\theta| + 2\cot\theta + 2\cos\theta = 2\cot\theta \left(\theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]\right)$

4. The number of real solutions of the equation $(\sin x - x)(\cos x - x^2) = 0$ is

(A) 1 (B) 2
(C) 3 (D) 4

Ans : (C)

Hint : $\sin x = x$ only $x = 0$ one solution

$\cos x = x^2$  2 solutions

5. Which of the following is not always true ?

(A) $|\vec{a}+\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2$ if $\vec{a}$ and $\vec{b}$ are perpendicular to each other

(B) $|\vec{a}+\lambda\vec{b}|\geq|\vec{a}|$ for all $\lambda \in R$ if $\vec{a}$ and $\vec{b}$ are perpendicular to each other

(C) $|\vec{a}+\vec{b}|^2+|\vec{a}-\vec{b}|^2=2\left(|\vec{a}|^2+|\vec{b}|^2\right)$

(D) $|\vec{a}+\lambda\vec{b}|\geq|\vec{a}|$ for all $\lambda \in R$ if $\vec{a}$ is parallel to $\vec{b}$

Ans : (D)

6. If the four points with position vectors $-2\hat{i}+\hat{j}+k,\ \hat{i}+\hat{j}+\hat{k},\ \hat{j}-\hat{k}$ and $\lambda\hat{j}+\hat{k}$ are coplanar, then $\lambda =$

(A) 1 (B) 2

(C) –1 (D) 0

Ans : (A)

Hint : Let $\vec{a}=-2\hat{i}+\hat{j}+\hat{k},\ \vec{b}=\hat{i}+\hat{j}+\hat{k},\ \vec{c}=\hat{j}-\hat{k},\ \vec{d}=\lambda\hat{j}+\hat{k}$

$\vec{b}-\vec{a}=3\hat{i},\ \vec{c}-\vec{b}=-\hat{i}-2\hat{k},\ \vec{d}-\vec{c}=(\lambda-1)\hat{j}+2\hat{k}$

$$\begin{vmatrix} 3 & 0 & 0 \\ -1 & 0 & -2 \\ 0 & \lambda-1 & 2 \end{vmatrix}=0 \Rightarrow \lambda = 1$$

7. For all real values of a_0, a_1, a_2, a_3 satisfying $a_0+\frac{a_1}{2}+\frac{a_2}{3}+\frac{a_3}{4}=0,$ the equation $a_0 + a_1x + a_2x^2 + a_3x^3 = 0$ has a real root in the interval

(A) [0, 1] (B) [–1, 0]

(C) [1, 2] (D) [–2, –1]

Ans : (A)

Hint : $f(x)=\frac{a_3x^4}{4}+\frac{a_2x^3}{3}+\frac{a_1x^2}{2}+a_0x$

$f(0) = f(1) = 0$

$\Rightarrow f'(x) = 0$ has one real root in [0, 1]

(Rolle's theorem)

8. Let $f : R \rightarrow R$ be defined as $f(x)=\begin{cases} 0, & x \text{ is irrational} \\ \sin|x|, & x \text{ is rational} \end{cases}$

Then which of the following is true ?

(A) f is discontinuous for all x (B) f is continuous for all x

(C) f is discontinuous at $x = k\pi$, where k is an integer (D) f is continuous at $x = k\pi$, where k is an integer

Ans : (D)

Hint : $\sin|x| = 0 \Rightarrow x = k\pi$

9. If $f:[0,\pi/2)\to R$ is defined as $f(\theta)=\begin{vmatrix}1 & \tan\theta & 1\\ -\tan\theta & 1 & \tan\theta\\ -1 & -\tan\theta & 1\end{vmatrix}$. Then the range of f is

(A) $(2,\infty)$ (B) $(-\infty,-2]$

(C) $[2,\infty)$ (D) $(-\infty,2]$

Ans : (C)

Hint : $f(\theta)=2\sec^2\theta$

Range of $f\to[2,\infty)$

10. If A and B are two matrices such that AB = B and BA = A, then A^2+B^2 equals

(A) 2AB (B) 2BA

(C) A + B (D) AB

Ans : (C)

Hint : A.A + B.B = A (BA) + B (AB) = BA + AB = A + B

11. If ω is an imaginary cube root of unity, then the value of the determinant $\begin{vmatrix}1+\omega & \omega^2 & -\omega\\ 1+\omega^2 & \omega & -\omega^2\\ \omega+\omega^2 & \omega & -\omega^2\end{vmatrix}$ is

(A) -2ω (B) $-3\omega^2$

(C) -1 (D) 0 (zero)

Ans : (B)

Hint : $\begin{vmatrix}1+\omega & \omega^2 & -\omega\\ 1+\omega^2 & \omega & -\omega^2\\ \omega+\omega^2 & \omega & -\omega^2\end{vmatrix}=\begin{vmatrix}0 & \omega^2 & -\omega\\ 0 & \omega & -\omega^2\\ -1+\omega & \omega & -\omega^2\end{vmatrix}$ $(c_1\to c_1+c_2)$

$=(-1+\omega)(-\omega^4+\omega^2)=(\omega-1)(\omega^2-\omega)=\omega^3-\omega^2-\omega^2+\omega=-3\omega^2$

12. Let a, b, c, d be any four real numbers. Then $a^n+b^n=c^n+d^n$ holds for any natural number n if

(A) a + b = c + d (B) a – b = c – d

(C) a + b = c + d, $a^2+b^2=c^2+d^2$ (D) a – b = c – d, $a^2-b^2=c^2-d^2$

Ans : (D)

Hint : Put n = 1, a + b = c + d ...... (1)

Put n = 3 $a^3+b^3=c^3+d^3$ .... (2)

from (1) and (2) ab = cd

Consider a quadratic with roots (a^3, b^3)

$x^2-(a^3+b^3)x+a^3b^3=0$ .... (3)

Consider another quadratic with roots (c^3, d^3)

$x^2-(c^3+d^3)x+(cd)^3=0$ .... (4)

Since $a^3+b^3=c^3+d^3$ and $(cd)^3=(ab)^3$

Both quadratic are same and quadratic cannot have more than 2 roots.

Here a = c and b = d or a = d, b = c

13. If α, β are the roots of $x^2 - px + 1 = 0$ and γ is a root of $x^2 + px + 1 = 0$, then $(\alpha + \gamma)(\beta + \gamma)$ is

(A) 0 (zero) (B) 1

(C) –1 (D) p

Ans : (A)

Hint : $\gamma = -\alpha$ or $-\beta$

$\Rightarrow (\alpha + \gamma)(\beta + \gamma) = 0$

14. Number of irrational terms in the binomial expansion of $\left(3^{1/5} + 7^{1/3}\right)^{100}$ is

(A) 90 (B) 88

(C) 93 (D) 95

Ans : (No option is correct)

Hint : $T_{r+1} = {}^{100}C_r (3)^{\frac{100-r}{5}} (7)^{\frac{r}{3}}$ r = 0, 15, 30, 45, 60, 75, 90 7 rational terms

No. of irrational terms

= 101 – 7 = 94 (not in the option).

15. The quadratic expression $(2x+1)^2 - px + q \neq 0$ for any real x if

(A) $p^2 - 16p - 8q < 0$ (B) $p^2 - 8p + 16q < 0$

(C) $p^2 - 8p - 16q < 0$ (D) $p^2 - 16p + 8q < 0$

Ans : (C)

Hint : $4x^2 + (4-p)x + q + 1 \neq 0$

$D < 0 \Rightarrow p^2 - 8p - 16q < 0$

16. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{64} + \left(\frac{1-\sqrt{3}i}{1+\sqrt{3}i}\right)^{64}$ is

(A) 0 (zero) (B) –1

(C) 1 (D) i

Ans : (B)

Hint : $1+\sqrt{3}i = -2\omega^2$

$1-\sqrt{3}i = -2\omega$

$\omega^{64} + \frac{1}{\omega^{64}} = \omega + \omega^2 = -1$

17. Find the maximum value of |z| when $\left|z - \frac{3}{z}\right| = 2$, z being a complex number.

(A) $1+\sqrt{3}$ (B) 3

(C) $1+\sqrt{2}$ (D) 1

Ans : (B)

Hint : $|z| - 3/|z| \leq 2 \Rightarrow |z|^2 - 2|z| - 3 \leq 0$ $|z|_{max} = 3$

18. Given that x is a real number satisfying $\frac{5x^2-26x+5}{3x^2-10x+3}<0$, then

(A) $x<\frac{1}{5}$

(B) $\frac{1}{5}<x<3$

(C) $x > 5$

(D) $\frac{1}{5}<x<\frac{1}{3}$ or $3 < x < 5$

Ans : (D)

Hint : $\frac{(x-5)\left(x-\frac{1}{5}\right)}{(x-3)\left(x-\frac{1}{3}\right)}<0$

$x \in \left(\frac{1}{5},\frac{1}{3}\right)\cup(3,5)$

19. The least positive value of t so that the lines $x = t + \alpha$, $y + 16 = 0$ and $y = \alpha x$ are concurrent is

(A) 2

(B) 4

(C) 16

(D) 8

Ans : (D)

Hint : $t = -\left(\frac{\alpha^2+16}{\alpha}\right)=-\left(\alpha+\frac{16}{\alpha}\right)$

least positive value = 8

20. If in a triangle ΔABC, $a^2\cos^2 A - b^2 - c^2 = 0$, then

(A) $\frac{\pi}{4}<A<\frac{\pi}{2}$

(B) $\frac{\pi}{2}<A<\pi$

(C) $A = \frac{\pi}{2}$

(D) $A < \frac{\pi}{4}$

Ans : (B)

Hint : $\cos^2 A = \frac{b^2+c^2}{a^2}$

$\cos^2 A < 1$ $b^2+c^2-a^2<0$ $\cos A < 0$ $A \in \left(\frac{\pi}{2},\pi\right)$

21. $\{x \in R : |\cos x| \geq \sin x\} \cap \left[0,\frac{3\pi}{2}\right] =$

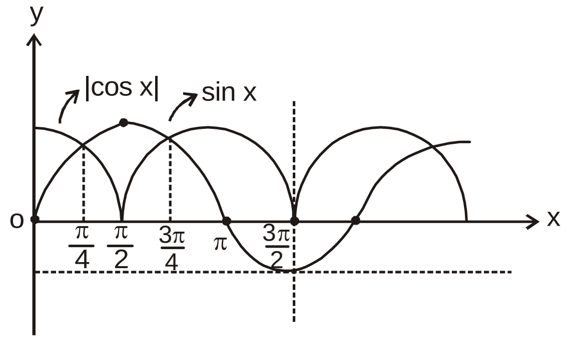
(A) $\left[0,\frac{\pi}{4}\right]\cup\left[\frac{3\pi}{4},\frac{3\pi}{2}\right]$

(B) $\left[0,\frac{\pi}{4}\right]\cup\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$

(C) $\left[0,\frac{\pi}{4}\right]\cup\left[\frac{5\pi}{4},\frac{3\pi}{2}\right]$

(D) $\left[0,\frac{3\pi}{2}\right]$

Ans : (A)

Hint : 

$$=\left[0,\frac{\pi}{4}\right]\cup\left[\frac{3\pi}{4},\frac{3\pi}{2}\right]$$

22. A particle starts moving from rest from a fixed point in a fixed direction. The distance s from the fixed point at a time t is given by $s = t^2 + at - b + 17$, where a, *b* are real numbers. If the particle comes to rest after 5 sec at a distance of s = 25 units from the fixed point, then values of a and b are respectively

(A) 10, –33 (B) –10, –33

(C) –8, 33 (D) –10, 33

Ans : (B)

Hint : $\left[\frac{ds}{dt}\right]_{t=5}=\left[2t+a\right]_{t=5}=0$

$\Rightarrow a = -10$

and $25 = 5^2 + 5a - b + 17$ (s=25, t=5)

$\Rightarrow b = -33$

23. $\lim\limits_{n\to\infty}\frac{\sqrt{1}+\sqrt{2}+.....+\sqrt{n-1}}{n\sqrt{n}}$ =0

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) 0(zero)

Ans : (C)

Hint : $\underset{n\to\infty}{\mathrm{lt}}\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^{\frac{1}{2}}-\underset{n\to\infty}{\mathrm{lt}}\frac{1}{n}\times\frac{n}{n}$

$$=\int_0^1\sqrt{x}\,dx+0=\frac{2}{3}$$

24. If $\lim\limits_{x\to 0}\frac{axe^x-b\log(1+x)}{x^2}=3$ then the values of *a*, *b* are respectively

(A) 2,2 (B) 1,2

(C) 2,1 (D) 2,0

Ans : (A)

Hint : (LH Rule)

$$\lim_{x\to 0}\frac{ae^{x}+axe^{x}-\frac{b}{1+x}}{2x} \quad \underline{\hspace{2cm}} (1)$$

(form $\frac{0}{0}$)

$a+0-b = 0 \Rightarrow a = b$ ——— (2)

(Again LH Rule in 1)

$$= a\left(\lim_{x\to 0}\frac{e^{x}(1+x)+e^{x}+\frac{1}{(1+x)^{2}}}{2}\right)$$

$$= \frac{3a}{2} \Rightarrow \frac{3a}{2} = 3 \Rightarrow a = 2$$

25. Let $P(x)$ be a polynomial, which when divided by $x-3$ and $x-5$ leaves remainders 10 and 6 respectively. If the polynomial is divided by $(x-3)(x-5)$ then the remainder is

(A) $-2x+16$ (B) 16

(C) $2x-16$ (D) 60

Ans : (A)

Hint : $\because$ $P(x) = (x-3)(x-5)\,q(x) + (ax+b)$

$\because$ $P(3) = 10$; $p(5) = 6$

$\Rightarrow 3a + b = 10;\ 5a + b = 6$

$\Rightarrow a = -2;\ b = 16$

$\therefore$ Remainder $= -2x + 16$

26. The integrating factor of the differential equation $\frac{dy}{dx} + (3x^2 \tan^{-1} y - x^3)(1+y^2) = 0$ is

(A) e^{x^2} (B) e^{x^3}

(C) e^{3x^2} (D) e^{3x^3}

Ans : (B)

Hint : $\tan^{-1} y = z \Rightarrow \frac{1}{1+y^2}\frac{dy}{dx} = \frac{dz}{dx}$

$$\Rightarrow \frac{dz}{dx} + 3x^2 . z = x^3$$

$\therefore$ I. F. $= e^{\int 3x^2 dx} = e^{x^3}$

27. If y = $e^{-x}\cos 2x$ then which of the following differential equations is satisfied ?

(A) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=0$ (B) $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+2y=0$

(C) $\frac{d^2y}{dx^2}-5\frac{dy}{dx}-2y=0$ (D) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}-5y=0$

Ans : (A)

Hint : $\therefore y_1 = -y - 2e^{-x}\sin 2x$ —— (1)

$\therefore y_2 = -y_1 + 2e^{-x}\sin 2x - 4e^{-x}\cos 2x$

$= -y_1 - y - y_1 - 4y$

$\Rightarrow y_2 + 2y_1 + 5y = 0$

28. In a certain town, 60% of the families own a car, 30% own a house and 20% own both a car and a house. If a family is randomly chosen, what is the probability that this family owns a car or a house but not both ?

(A) 0.5 (B) 0.7

(C) 0.1 (D) 0.9

Ans : (A)

Hint : A = Car ; B = House

$P(A\Delta B) = P(A\cup B) - P(A\cap B)$

$= \frac{70-20}{100} = 0.5$

29. The letters of the word COCHIN are permuted and all the permutations are arranged in alphabetical order as in English dictionary. The number of words that appear before the word COCHIN is

(A) 360 (B) 192

(C) 96 (D) 48

Ans : (C)

Hint : Rank of COCHIN = (4!)4 + 1 = 97 $\therefore$ Number of words = 97–1 = 96

30. Let $f : R\to R$ a continuous function which satisfies $f(x)=\int_0^x f(t)\,dt$. Then the value of $f(\log_e 5)$ is

(A) 0 (zero) (B) 2

(C) 5 (D) 3

Ans : (A)

Hint : $f'(x) = f(x)$ (Leibnitz theorem)

$\Rightarrow f(x) = ke^x$ and $f(0)=\int_0^0 f(t)\,dt = 0 \Rightarrow k = 0 \therefore f(x) = 0 \Rightarrow f(\log_e 5) = 0$

31. The value of λ, such that the following system of equations has no solution, is

$2x - y - 2z = 2$

$x - 2y + z = -4$

$x + y + \lambda z = 4$

(A) 3 (B) 1

(C) 0(zero) (D) –3

Ans : (D)

Hint : $\begin{vmatrix} 2 & -1 & -2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0 \Rightarrow -3\lambda - 9 = 0 \Rightarrow \lambda = -3$

32. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$

Then $f(100)$ is equal to

(A) 0(zero) (B) 1

(C) 100 (D) 10

Ans : (A)

Hint : $f(x) = x(x+1)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x-1 & x \\ 3x(x-1) & (x-1)(x-2) & x(x-1) \end{vmatrix}$

$$C_1 \sim C_1 - C_3 \text{ ; } C_2 \sim C_2 - C_3$$

$= x(x+1)\begin{vmatrix} 0 & 0 & 1 \\ x & -1 & x \\ 2x(x-1) & -2(x-1) & x(x-1) \end{vmatrix}$

$= x(x+1)\begin{vmatrix} x & -1 \\ 2x(x-1) & -2(x-1) \end{vmatrix} = 0$

$\therefore f(100) = 0$

33. If $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \frac{x^4}{8} + \ldots\right) = \frac{\pi}{6}$ where $|x| < 2$ then the value of x is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$

(C) $-\frac{2}{3}$ (D) $-\frac{3}{2}$

Ans : (A)

Hint : $\sin^{-1}\left(\frac{x}{1-\left(-\frac{x}{2}\right)}\right) = \frac{\pi}{6}$ $[\because |x| < 2 \Rightarrow \frac{|x|}{2} < 1]$

$\Rightarrow \frac{2x}{2+x} = \frac{1}{2} \Rightarrow x = \frac{2}{3}$

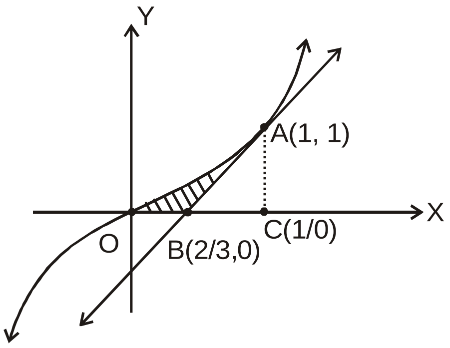
34. The area of the region bounded by the curve $y = x^3$, its tangent at (1, 1) and x-axis is

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$

(C) $\frac{2}{17}$ (D) $\frac{2}{15}$

Ans : (A)

Hint :



Equation of tangent at A is $y = 3x - 2$.

Area of shaded region $= \int_0^1 x^3 dx - \int_{2/3}^1 (3x-2)\,dx = \frac{1}{4} - \frac{15}{18} + \frac{2}{3} = \frac{1}{12}$

35. If $\log_{0\ 2}(x-1) > \log_{0\ 04}(x+5)$ then

(A) $-1 < x < 4$ (B) $2 < x < 3$

(C) $1 < x < 4$ (D) $1 < x < 3$

Ans : (C)

Hint : $\log_{0.2}(x-1) > \log_{0.2^2}(x+5) \Rightarrow \log_{0.2}(x-1) > \frac{1}{2}\log_{0.2}(x+5)$

$\Rightarrow (x-1)^2 < (x+5) \Rightarrow x^2 - 3x - 4 < 0 \Rightarrow -1 < x < 4$ (but $x > 1$)

$\Rightarrow 1 < x < 4$

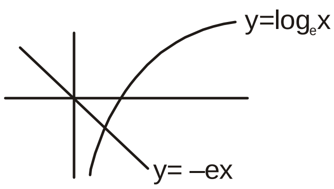
36. The number of real roots of equation $\log_e x + ex = 0$

(A) 0(zero) (B) 1

(C) 2 (D) 3

Ans : (B)

Hint :



Clearly one root is there

37. If the vertex of the conic $y^2 - 4y = 4x - 4a$ always lies between the straight lines $x + y = 3$ and $2x + 2y - 1 = 0$ then

(A) $2 < a < 4$
(B) $-\frac{1}{2} < a < 2$
(C) $0 < a < 2$
(D) $-\frac{1}{2} < a < \frac{3}{2}$

Ans : (B)

Hint : Vertex of $y^2 - 4y = 4x - 4a$ is $(a-1, 2)$

So, $(a-1 + 2 - 3)(2a - 2 + 4 - 1) < 0$

$(a-2)(2a + 1) < 0$

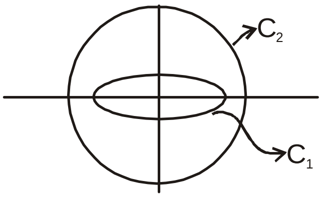
$-\frac{1}{2} < a < 2$

38. Number of intersecting points of the conic $4x^2 + 9y^2 = 1$ and $4x^2 + y^2 = 4$ is

(A) 1
(B) 2
(C) 3
(D) 0(zero)

Ans : (D)

Hint : $C_1 : \frac{x^2}{1/4} + \frac{y^2}{1/9} = 1 \quad C_2 : \frac{x^2}{1} + \frac{y^2}{4} = 1$



Clearly no intersection

39. The value of λ for which the straight line $\frac{x-\lambda}{3} = \frac{y-1}{2+\lambda} = \frac{z-3}{-1}$ may lie on the plane $x - 2y = 0$ is

(A) 2
(B) 0
(C) $-\frac{1}{2}$
(D) there is no such λ

Ans : (C)

Hint : The line must be perpendicular to the normal to plane.

$3.1 + (2 + \lambda)(-2) + (-1).0 = 0$

$\lambda = -1/2$

40. The value of $2\cot^{-1}\frac{1}{2} - \cot^{-1}\frac{4}{3}$ is

(A) $-\frac{\pi}{8}$
(B) $\frac{3\pi}{2}$
(C) $\frac{\pi}{4}$
(D) $\frac{\pi}{2}$

Ans : (D)

Hint : $2\tan^{-1}2 - \tan^{-1}\frac{3}{4}$

$= \pi + \tan^{-1}\left(-\frac{4}{3}\right) - \tan^{-1}\frac{3}{4}$

$= \pi - \left(\tan^{-1}\frac{4}{3} + \tan^{-1}\frac{3}{4}\right)$

$= \pi - \frac{\pi}{2} = \frac{\pi}{2}$

41. If the point $(2\cos\theta, 2\sin\theta)$, for $\theta \in (0, 2\pi)$ lies in the region between the lines $x + y = 2$ and $x - y = 2$ containing the origin, then θ lies in

(A) $\left(0,\frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2},2\pi\right)$

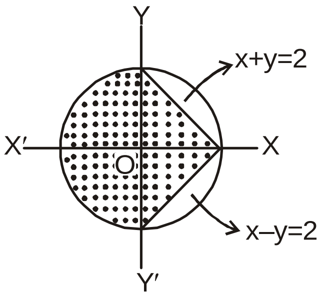
(B) $[0,\pi]$

(C) $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$

(D) $\left[\frac{\pi}{4},\frac{\pi}{2}\right]$

Ans : (C)

Hint :



$(2\cos\theta, 2\sin\theta)$ will lie on the circle, $x^2 + y^2 = 4$, for the marked region in the figure. So, $\pi/2 < \theta < 3\pi/2$

42. Number of points having distance $\sqrt{5}$ from the straight line $x-2y + 1 = 0$ and a distance $\sqrt{13}$ from the line $2x + 3y - 1 = 0$ is

(A) 1

(B) 2

(C) 4

(D) 5

Ans : (C)

Hint : Let the point be (a, b)

So, $\frac{|a-2b+1|}{\sqrt{5}} = \sqrt{5} \Rightarrow a - 2b + 1 = \pm 5$ .... (1)

and $\frac{|2a+3b-1|}{\sqrt{13}} = \sqrt{13} \Rightarrow 2a + 3b - 1 = \pm 13$ ....(2)

Any one of equations (1) combined with any one of equations (2) will yield a point $\Rightarrow$ 4 points

43. Let f : R $\rightarrow$ R be defined as $f(x)=\dfrac{x^2-x+4}{x^2+x+4}$. Then the range of the function f(x) is

(A) $\left[\dfrac{3}{5},\dfrac{5}{3}\right]$ (B) $\left(\dfrac{3}{5},\dfrac{5}{3}\right)$ (C) $\left(-\infty,\dfrac{3}{5}\right)\cup\left(\dfrac{5}{3},\infty\right)$ (D) $\left[-\dfrac{5}{3},-\dfrac{3}{5}\right]$

Ans : (A)

Hint : Let, $y=\dfrac{x^2-x+4}{x^2+x+4}\Rightarrow (y-1)x^2+(y+1)x+4y-4=0$

For x to be real, discriminant of the above quadratic equation should be greater than or equal to zero

$\Rightarrow \dfrac{3}{5}\le y\le\dfrac{5}{3}$

($y-1=0\Rightarrow y=1\Rightarrow x=0$ which is valid)

44. The least value of $2x^2+y^2+2xy+2x-3y+8$ for real numbers x and y is

(A) 2 (B) 8 (C) 3 (D) 1

Ans : (No option is correct)

Hint : $2x^2+y^2+2xy+2x-3y+8$

$=\dfrac{1}{2}(4x^2+2y^2+4xy+4x-6y+16)$

$=\dfrac{1}{2}\left\{(y-4)^2+(2x+y+1)^2-1\right\}\ge-\dfrac{1}{2}$

so, least value is $-\dfrac{1}{2}$ at $x=-\dfrac{5}{2},y=4$.

Correct option is not there.

45. Let f : [– 2, 2] $\rightarrow$ R be a continuous function such that f(x) assumes only irrational values. If $f\left(\sqrt{2}\right)=\sqrt{2}$, then

(A) $f(0)=0$ (B) $f\left(\sqrt{2}-1\right)=\sqrt{2}-1$

(C) $f\left(\sqrt{2}-1\right)=\sqrt{2}+1$ (D) $f\left(\sqrt{2}-1\right)=\sqrt{2}$

Ans : (D)

Hint : A continuous function assuming only irrational value must be constant function

So, $f(x)=\sqrt{2}$

46. The minimum value of $\cos\theta+\sin\theta+\dfrac{2}{\sin 2\theta}$ for $\theta\in(0,\pi/2)$ is

(A) $2+\sqrt{2}$ (B) 2 (C) $1+\sqrt{2}$ (D) $2\sqrt{2}$

Ans : (A)

Hint : $\sin\theta+\cos\theta+\dfrac{1}{\sin\theta.\cos\theta}$

As the expression remain unchanged by interchanging $\sin\theta$ and $\cos\theta$ so minimum is achieved for

$\sin\theta=\cos\theta$ so $\theta=\dfrac{\pi}{4}$ in $\left(0,\dfrac{\pi}{2}\right)$

so, minimum value = $2+\sqrt{2}$

47. Let $x_n = \left(1-\frac{1}{3}\right)^2\left(1-\frac{1}{6}\right)^2\left(1-\frac{1}{10}\right)^2 \ldots\ldots\ldots \left(1-\frac{1}{\frac{n(n+1)}{2}}\right)^2, n \geq 2.$

Then the value of $\lim_{n\to\infty} x_n$ is

(A) 1/3 (B) 1/9 (C) 1/81 (D) 0 (zero)

Ans : (B)

Hint : $\frac{x_n}{x_{n-1}} = \left(1-\frac{2}{n(n+1)}\right)^2 = \frac{\left(\frac{n+2}{n}\right)^2}{\left(\frac{n+1}{n-1}\right)^2}$

so comparing $x_n = A.\left(\frac{n+2}{n}\right)^2$, A is real constant,

now, $x_2 = \frac{4}{9}$. So $A = \frac{1}{9}$.

so $\lim_{n\to\infty} x_n = A = \frac{1}{9}$

48. The variance of first 20 natural numbers is

(A) 133/4 (B) 279/12 (C) 133/2 (D) 399/4

Ans : (A)

Hint : Variance of first n natural numbers is $\frac{n^2-1}{12}$

so, for n = 20 $\to \frac{20^2-1}{12} = \frac{133}{4}$

49. A fair coin is tossed a fixed number of times. If the probability of getting exactly 3 heads equals the probability of getting exactly 5 heads, then the probability of getting exactly one head is

(A) 1/64 (B) 1/32 (C) 1/16 (D) 1/8

Ans : (B)

Hint : Let, the fixed number is n.

So, ${}^nc_3 = {}^nc_5 \Rightarrow n = 8$

Thus, required probability is ${}^8c_1 \times \left(\frac{1}{2}\right)^8 = \frac{1}{32}$

50. If the letters of the word PROBABILITY are written down at random in a row, the probability that two B-s are together is

(A) 2/11 (B) 10/11 (C) 3/11 (D) 6/11

Ans : (A)

Hint : $\frac{\frac{10!}{2!}}{\frac{11!}{2!2!}} = \frac{2}{11}$

51. The number of distinct real roots of

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0 \text{ in the interval } -\frac{\pi}{4} \le x \le \frac{\pi}{4} \text{ is}$$

(A) 0 (zero) (B) 2

(C) 1 (D) >2

Ans : (C)

Hint : Determinant = (sin x + 2 cos x) (sin x – cos x)^2=0

so, either tan x = –2 or tan x = 1

only one solution

52. Let x_1, x_2, ..., x_{15} be 15 distinct numbers chosen from 1, 2, 3, ..., 15. Then the value of $(x_1 - 1)$ $(x_2 - 1)$, $(x_3 - 1)$............ $(x_{15} - 1)$ is

(A) always ≤ 0 (B) 0 (zero) (C) always even (D) always odd

Ans : (B)

Hint : Among x_1, x_2, x_3, ... , x_{15}; one of them, say, x_k is 1.

$\Rightarrow x_k - 1 = 0 \Rightarrow$ the product given is 0

53. Let [x] denote the greatest integer less than or equal to x Then the value of α for which the function

$$f(x) = \begin{cases} \dfrac{\sin[-x^2]}{[-x^2]}, & x \ne 0 \\ \alpha, & x = 0 \end{cases} \text{ is continuous at } x = 0 \text{ is}$$

(A) $\alpha = 0$ (B) $\alpha = \sin(-1)$ (C) $\alpha = \sin(1)$ (D) $\alpha = 1$

Ans : (C)

Hint : $\lim\limits_{x \to 0} \dfrac{\sin[-x^2]}{[-x^2]} = \sin(1)$

if $f(x)$ is continuous at x = 0, then $\lim\limits_{x \to 0} f(x) = f(0) = \alpha = \sin(1)$

54. Let f(x) denote the fractional part of a real number x. Then the value of $\int_0^{\sqrt{3}} f(x^2)dx$

(A) $2\sqrt{3} - \sqrt{2} - 1$ (B) 0 (zero)

(C) $\sqrt{2} - \sqrt{3} + 1$ (D) $\sqrt{3} - \sqrt{2} + 1$

Ans : (C)

Hint : $\int_0^{\sqrt{3}} f(x^2)dx = \int_0^{\sqrt{3}} \{x^2\}dx = \int_0^{\sqrt{3}} \left(x^2 - [x^2]\right)dx = \int_0^{\sqrt{3}} x^2 dx - \left(\int_0^1 [x^2]dx + \int_1^{\sqrt{2}} [x^2]dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2]dx\right)$

$= \sqrt{3} - \left(\int_0^1 0.dx + \int_1^{\sqrt{2}} 1.dx + \int_{\sqrt{2}}^{\sqrt{3}} 2.dx\right) = \sqrt{3} - \left(\sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2}\right) = \sqrt{2} - \sqrt{3} + 1$

55. Let S = {(a, b, c) $\in$ N × N × N : a + b + c = 21, a $\le$ b $\le$ c} and

T = {(a, b, c) $\in$ N × N × N : a, b, c are in A.P.}, where *N* is the set of all natural numbers. Then the number of elements in the set S $\cap$ T is

(A) 6 (B) 7

(C) 13 (D) 14

Ans : (B)

Hint : a + b + c = 21 and $b = \frac{a+c}{2}$

$\Rightarrow$ a + c = 14 and b = 7

So, a can take values from 1 to 6, when c ranges from 13 to 8, or a = b = c = 7

So, 7 triplets

56. Let $y = e^{x^2}$ and $y = e^{x^2}\sin x$ be two given curves. Then the angle between the tangents to the curves at any point of their intersection is

(A) 0 (zero) (B) π

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

Ans : (A)

Hint : $e^{x^2} = e^{x^2}\sin x$ $\Rightarrow$ sin x = 1 at point of intersection.

$y = e^{x^2} \Rightarrow \frac{dy}{dx} = 2x\,e^{x^2}$

Again, $y = e^{x^2}\sin x \Rightarrow \frac{dy}{dx} = 2xe^{x^2}\sin x + e^{x^2}\cos x$

$\Rightarrow \frac{dy}{dx} = 2xe^{x^2}$ (when sin x = 1 i.e. cos x = 0)

So, slopes of tangents are equal at point of intersection

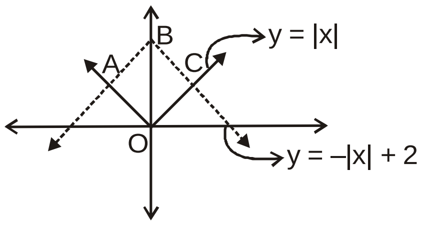
57. Area of the region bounded by y = |x| and y = –|x| + 2 is

(A) 4 sq. units (B) 3 sq. units

(C) 2 sq. units (D) 1 sq. units

Ans : (C)

Hint :



In figure, B $\equiv$ (0, 2) $\Rightarrow$ OB = 2 and OABC is square.

So, side is of length $\sqrt{2}$ units $\Rightarrow$ area OABC = 2 sq. units.

58. Let d(n) denote the number of divisors of n including 1 and itself. Then d(225), d(1125) and d(640) are

(A) in AP
(B) in HP
(C) in GP
(D) consecutive integers

Ans : (C)

Hint : $225 = 3^2 \times 5^2 \Rightarrow d(225) = 3 \times 3 = 9$

$1125 = 3^2 \times 5^3 \Rightarrow d(1125) = 3 \times 4 = 12$

$640 = 2^7 \times 5 \Rightarrow d(640) = 8 \times 2 = 16$

9, 12, 16 are in G.P.

59. The trigonometric equation $\sin^{-1}x = 2\sin^{-1}2a$ has a real solution if

(A) $|a| > \frac{1}{\sqrt{2}}$
(B) $\frac{1}{2\sqrt{2}} < |a| < \frac{1}{\sqrt{2}}$
(C) $|a| > \frac{1}{2\sqrt{2}}$
(D) $|a| \le \frac{1}{2\sqrt{2}}$

Ans : (D)

Hint : $-\pi/2 \le \sin^{-1}x \le \pi/2 \Rightarrow -\pi/2 \le 2\sin^{-1}2a \le \pi/2$

$\Rightarrow -\pi/4 \le \sin^{-1}2a \le \pi/4 \Rightarrow -\frac{1}{\sqrt{2}} \le 2a \le \frac{1}{\sqrt{2}}$

$\Rightarrow -\frac{1}{2\sqrt{2}} \le a \le \frac{1}{2\sqrt{2}} \Rightarrow |a| \le \frac{1}{2\sqrt{2}}$

60. If $2 + i$ and $\sqrt{5} - 2i$ are the roots of the equation $(x^2 + ax + b)(x^2 + cx + d) = 0$, where a, b, c, d are real constants, then product of all roots of the equation is

(A) 40
(B) $9\sqrt{5}$
(C) 45
(D) 35

Ans : (C)

Hint : $2 - i$ and $\sqrt{5} + 2i$ are other roots.

So, Product is $= (2 + i)(2 - i)\left(\sqrt{5} + 2i\right)\left(\sqrt{5} - 2i\right)$

$= 5 \times 9 = 45$

CATEGORY - II (Q61 to Q70)

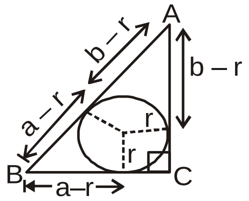
Each question has one correct option and carries 2 marks, for each wrong answer 1/2 mark will be deducted.

61. In a triangle ABC, $\angle C = 90°$, r and R are the in-radius and circum-radius of the triangle ABC respectively, then $2(r + R)$ is equal to

(A) b + c
(B) c + a
(C) a + b
(D) a + b + c

Ans : (C)

Hint :



$a - r + b - r = 2R$

$\Rightarrow 2(r + R) = a + b$

62. Let α, β be two distinct roots of a $\cos\theta$ + b $\sin\theta$ = c, where *a, b* and *c* are three real constants and $\theta \in [0, 2\pi]$. Then $\alpha + \beta$ is also a root of the same equation if

(A) a + b = c

(B) b + c = a

(C) c + a = b

(D) c = a

Ans : (D)

Hint : $$a\left(\frac{1-\tan^2 \theta/2}{1+\tan^2 \theta/2}\right)+\frac{2b\tan\theta/2}{1+\tan^2\theta/2}=c$$

$\Rightarrow$ (c + a) $\tan^2\theta/2$ – $2b\tan\theta/2$ + (c – a) = 0

so, $$\tan\frac{\alpha+\beta}{2}=\frac{\frac{2b}{c+a}}{1-\frac{c-a}{c+a}}=\frac{b}{a}$$

$\frac{b}{a}$ is a root of the equation,

$$(c+a)\frac{b^2}{a^2}-2b\left(\frac{b}{a}\right)+c-a=0$$

so, c = a

63. For a matrix

$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$, if U_1, U_2 and U_3 are 3 × 1 column matrices satisfying

$AU_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $AU_2 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$, $AU_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ and U is 3 × 3 matrix whose columns are U_1, U_2 and U_3

Then sum of the elements of U^{-1} is

(A) 6

(B) 0 (zero)

(C) 1

(D) 2/3

Ans : (B)

Hint : Let $U_i = \begin{pmatrix} a_i \\ b_i \\ c_i \end{pmatrix}$ $i=1,2,3$

$AU_1 = \begin{pmatrix} a_1 \\ 2a_1+b_1 \\ 3a_1+2b_1+c_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$**,** So $a_1 = 1$, $b_1 = -2$, $c_1 = 1$

$AU_2 = \begin{pmatrix} a_2 \\ 2a_2+b_2 \\ 3a_2+2b_2+c_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$,

So, $a_2 = 2$, $b_2 = -1$, $c_2 = -4$**.** Similarly, $a_3 = 2$, $b_3 = -1$, $c_3 = -3$

So, $U = \begin{pmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{pmatrix}$. So, sum of elements of U^{-1} is zero.

64. Let $f : N \to R$ be such that $f(1) = 1$ and
$f(1) + 2f(2) + 3f(3) + \ldots\ldots + nf(n) = n(n+1)f(n)$, for all $n \in N$, $n \geq 2$, where N is the set of natural numbers and R is the set of real numbers. Then the value of $f(500)$ is

(A) 1000 (B) 500

(C) 1/500 (D) 1/1000

Ans : (D)

Hint : $f(1) + 2f(2) + 3f(3) + \ldots\ldots + nf(n) = n(n+1)f(n)$

$f(1) + 2f(2) + 3f(3) + \ldots\ldots + (n-1)f(n-1) = (n-1)nf(n-1)$

Subtracting, $nf(n) = n(n+1)f(n) - n(n-1)f(n-1)$

$\Rightarrow nf(n) = (n-1)f(n-1)$

Clearly, $f(n) = \frac{1}{2n}$. So, $f(500) = \frac{1}{1000}$

65. If 5 distinct balls are placed at random into 5 cells, then the probability that exactly one cell remains empty is

(A) 48/125 (B) 12/125

(C) 8/125 (D) 1/125

Ans : (A)

Hint : Required probability = $\frac{{}^5C_1\left(4^5 - {}^4C_1 \cdot 3^5 + {}^4C_2 \cdot 2^5 - {}^4C_3 \cdot 1^5\right)}{5^5} = \frac{48}{125}$

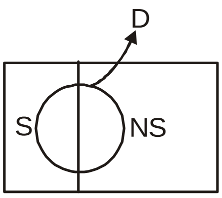
66. A survey of people in a given region showed that 20% were smokers. The probability of death due to lung cancer, given that a person smoked, was 10 times the probability of death due to lung cancer, given that a person did not smoke. If the probability of death due to lung cancer in the region is 0.006, what is the probability of death due to lung cancer given that a person is a smoker?

(A) 1/140 (B) 1/70

(C) 3/140 (D) 1/10

Ans : (C)

Hint :



S = person is smoker

NS = person is non smoker

D = death due to lung cancer

$$P(D) = P(S).\, P\left(\frac{D}{S}\right) + P(NS).P\left(\frac{D}{NS}\right)$$

$$0.006 = \frac{20}{100} \times P\left(\frac{D}{S}\right) + \frac{80}{100} \times \frac{1}{10} \times P\left(\frac{D}{S}\right)$$

$$\Rightarrow P\left(\frac{D}{S}\right) = \frac{3}{140}.$$

67. A person goes to office by a car or scooter or bus or train, probability of which are 1/7, 3/7, 2/7 and 1/7 respectively. Probability that he reaches office late, if he takes car, scooter, bus or train is 2/9, 1/9, 4/9, and 1/9 respectively. Given that he reached office in time, the probability that he travelled by a car is

(A) 1/7 (B) 2/7

(C) 3/7 (D) 4/7

Ans : (A)

Hint : Required probability = $\dfrac{\frac{1}{7}\times\frac{7}{9}}{\frac{1}{7}\times\frac{7}{9}+\frac{3}{7}\times\frac{8}{9}+\frac{2}{7}\times\frac{5}{9}+\frac{1}{7}\times\frac{8}{9}}=\dfrac{1}{7}$

68. The value of $\int\dfrac{(x-2)dx}{\left\{(x-2)^2(x+3)^7\right\}^{1/3}}$ is

(A) $\dfrac{3}{20}\left(\dfrac{x-2}{x+3}\right)^{4/3}+c$ (B) $\dfrac{3}{20}\left(\dfrac{x-2}{x+3}\right)^{3/4}+c$

(C) $\dfrac{5}{12}\left(\dfrac{x-2}{x+3}\right)^{4/3}+c$ (D) $\dfrac{3}{20}\left(\dfrac{x-2}{x+3}\right)^{5/3}+c$

Ans : (A)

Hint : I = $\int\dfrac{dx}{(x-2)^{-1/3}\cdot(x+3)^{7/3}}=\int\dfrac{dx}{(x-2)^2\cdot\left(\dfrac{x+3}{x-2}\right)^{7/3}}$

Let, $\dfrac{x+3}{x-2}$ = t so, $\dfrac{-5dx}{(x-2)^2}$ = dt

So, I = $-\dfrac{1}{5}\int\dfrac{dt}{t^{7/3}}=\dfrac{3}{20t^{4/3}}+c\ =\ \dfrac{3}{20}\cdot\left(\dfrac{x-2}{x+3}\right)^{4/3}+c$

69. Let $f: R\to R$ be differentiable at x = 0. If $f(0) = 0$ and $f'(0) = 2$, then the value of

$\lim\limits_{x\to0}\dfrac{1}{x}\left[f(x)+f(2x)+f(3x)+.....+f(2015x)\right]$ is

(A) 2015 (B) 0 (zero)

(C) 2015 × 2016 (D) 2015 × 2014

Ans : (C)

Hint : Applying L' Hospital Rule,

$$\lim_{x\to0}\frac{f'(x)+2f'(2x)+3f'(3x)+......+2015f'(2015x)}{1}=\frac{2\times2015\times2016}{2}=2015\times2016$$

70. If x and y are digits such that 17! = 3556xy428096000, then x + y equals

(A) 15 (B) 6

(C) 12 (D) 13

Ans : (A)

Hint : Since 17! is divisible by 9 so sum of the digits 48 + x + y must be divisible by 9.

So, x + y can be 15 or 6.

Also 17! is divisible by 11 so |10 + x – y| must be multiple of 11 or zero. The only possibility is |x – y| = 1
So, x + y = 15

CATEGORY - III (Q71 to Q80)

Each question has one or more correct option(s), choosing which will fetch maximum 2 marks on pro rata basis. However, choice of any wrong option(s) will fetch zero mark for the question.

71. Which of the following is/are always false?

(A) A quadratic equation with rational coefficients has zero or two irrational roots

(B) A quadratic equation with real coefficients has zero or two non-real roots

(C) A quadratic equation with irrational coefficients has zero or two rational roots

(D) A quadratic equation with integer coefficients has zero or two irrational roots

Ans : (C)

72. If the straight line $(a-1)x - by + 4 = 0$ is normal to the hyperbola $xy = 1$ then which of the followings does not hold?

(A) $a > 1, b > 0$ (B) $a > 1, b < 0$

(C) $a < 1, b < 0$ (D) $a < 1, b > 0$

Ans : (A,C)

Hint : Every normal to $xy = 1$ must have positive slope as $\frac{-dx}{dy} = x^2$. So $\frac{a-1}{b} > 0$.

73. Suppose a machine produces metal parts that contain some defective parts with probability 0.05. How many parts should be produced in order that probability of at least one part being defective is 1/2 or more? (Given $\log_{10}95 = 1.977$ and $\log_{10} 2 = 0.3$)

(A) 11 (B) 12

(C) 15 (D) 14

Ans : (A,B)

Hint : Probability of at least one part being defective

= 1 – probability of no part defective

= $1-(0.95)^n \geq \frac{1}{2}$

so $(0.95)^n \leq \frac{1}{2}$

$n[\log_{10} 95 - 2] \leq \log_{10} \frac{1}{2}$

$n \leq \frac{300}{23}$

so n = 11, 12

74. Let $f : R \to R$ be such that $f(2x-1) = f(x)$ for all $x \in R$. If f is continuous at $x = 1$ and $f(1) = 1$, then

(A) $f(2) = 1$ (B) $f(2) = 2$

(C) f is continuous only at $x = 1$ (D) f is continuous at all points

Ans : (A,D)

Hint : $x \to \left(\frac{x+1}{2}\right)$

$$f(x) = f\left(\frac{x+1}{2}\right) = f\left(\frac{\frac{x+1}{2}+1}{2}\right) = f\left(\frac{x+1+2}{2^2}\right)$$

$$= f\left(\frac{x+1+2+2^2+2^3+...2^{n-1}}{2^n}\right)$$

$$= f\left(\frac{x}{2^n}+\frac{2^n-1}{2^n}\right)$$

$$= f\left(\frac{x}{2^n}+1-\frac{1}{2^n}\right)$$

Taking limit $n \to \infty$,

$f(x) = f(1)$, as $f(x)$ is continuous at x = 1

$\Rightarrow$ $f(x)$ is constant function

75. If cos x and sin x are solutions of the differential equation $a_0\frac{d^2y}{dx^2}+a_1\frac{dy}{dx}+a_2y=0$,

Where a_0, a_1, a_2 are real constants then which of the followings is/are always true?

(A) A cos x + B sin x is a solution, where A and B are real constants

(B) $A\cos\left(x+\frac{\pi}{4}\right)$ is a solution, where A is real constant

(C) A cos x sin x is a solution, where A is real constant

(D) $A\cos\left(x+\frac{\pi}{4}\right)+B\sin\left(x-\frac{\pi}{4}\right)$ is a solution, where A and B are real constants

Ans : (A,B,D)

Hint : The general solution is $y = C_1 \cos x + C_2 \sin x$

where C_1 and C_2 are real constants

76. Which of the following statements is/are correct for $0<\theta<\frac{\pi}{2}$?

(A) $(\cos\theta)^{1/2} \le \cos\frac{\theta}{2}$

(B) $(\cos\theta)^{3/4} \ge \cos\frac{3\theta}{4}$

(C) $\cos\frac{5\theta}{6} \ge (\cos\theta)^{5/6}$

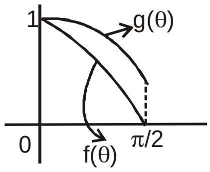
(D) $\cos\frac{7\theta}{8} \le (\cos\theta)^{7/8}$

Ans : (A,C)

Hint : $f(\theta)=\cos^n\theta \;\&\; g(\theta)=\cos n\theta,\ 0<\theta<\frac{\pi}{2}$

as $0 < n < 1$, $0<n\theta<\frac{\pi}{2}$. So $\cos n\theta$ ranges from 1 to some positve value whereas $\cos^n\theta$ ranges from 1 to 0.

Also $f'(\theta)<g'(\theta)$.



So, $\cos^{\frac{1}{2}}\theta \le \cos\frac{\theta}{2}$ and $\cos^{\frac{5}{6}}\theta \le \cos\frac{5\theta}{6}$.

77. Let $16x^2 - 3y^2 - 32x - 12y = 44$ represent a hyperbola. Then

(A) length of the transverse axis is $2\sqrt{3}$

(B) length of each latus rectum is $32/\sqrt{3}$

(C) eccentricity is $\sqrt{19/3}$

(D) equation of a directrix is $x = \frac{\sqrt{19}}{3}$

Ans : (A,B,C)

Hint : $16(x-1)^2 - 3(y+2)^2 = 48$

so, $\frac{(x-1)^2}{3} - \frac{(y+2)^2}{16} = 1$

Length of transverse axis $= 2\sqrt{3}$

Length of $L.R = {}^{32}\!/\!_{\sqrt{3}}$.

eccentricity $= \frac{\sqrt{19}}{\sqrt{3}}$.

equation of directrix, $x = 1 + \frac{3}{\sqrt{19}}$

78. For the function $f(x) = \left[\frac{1}{[x]}\right]$, where [x] denotes the greatest integer less than or equal to x, which of the following statements are true?

(A) The domain is $(-\infty, \infty)$

(B) The range is $\{0\} \cup \{-1\} \cup \{1\}$

(C) The domain is $\{-\infty, 0\} \cup [1, \infty)$

(D) The range is $\{0\} \cup \{1\}$

Ans : (B,C)

Hint : Domain is $= \mathbb{R} - [0,1)$

Range is = {1, –1, 0}

79. Let f be any continuously differentiable function on [a,b] and twice differentiable on (a,b) such that $f(a)=f'(a)=0$ and $f(b)=0$. Then

(A) $f''(a) = 0$

(B) $f'(x) = 0$ for some $x \in (a,b)$

(C) $f''(x) = 0$ for some $x \in (a,b)$

(D) $f'''(x) = 0$ for some $x \in (a,b)$

Ans : (B,C)

Hint : Applying Rolle's Theorem on f(x)

$f(a) = f(b) = 0$ so $f'(x) = 0$ for some $x = c \in (a,b)$ again $f'(a) = 0 = f'(c)$ so for some $x \in$

(a,c) i.e (a,b) $f''(x) = 0$.

80. A relation ρ on the set of real number R is defined as follows:

$x\rho y$ if and only if $xy > 0$. Then which of the followings is/are true?

(A) ρ is reflexive and symmetric

(B) ρ is symmetric but not reflexive

(C) ρ is symmetric and transitive

(D) ρ is an equivalence relation

Ans : (B,C)

Hint : $x \rho x : x^2 > 0$ not true for $x = 0$

$x\rho y \Rightarrow y\rho x$

$x \rho y$ so $xy > 0$, $y \rho z$ so $yz > 0$.

$xzy^2 > 0$, Hence $xz > 0$ so $x \rho z$.