Numbers-Key Notes

Divisibility

1. A number is divisible by 2 if it is an even number.

2. A number is divisible by 3 if the sum of the digits is divisible by 3.

3. A number is divisible by 4 if the number formed by the last two digits is divisible by 4.

4. A number is divisible by 5 if the units digit is either 5 or 0.

5. A number is divisible by 6 if the number is divisible by both 2 and 3.

6. A number is divisible by 8 if the number formed by the last three digits is divisible by 8.

7. A number is divisible by 9 if the sum of the digits is divisible by 9.

8. A number is divisible by 10 if the units digit is 0.

9. A number is divisible by 11 if the difference of the sum of its digits at odd places and the sum of its digits at even places, is divisible by 11.

Important formulas

i. $(a + b)(a - b) = (a^2 - b^2)$

ii. $(a + b)^2 = (a^2 + b^2 + 2ab)$

iii. $(a - b)^2 = (a^2 + b^2 - 2ab)$

iv. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2 (ab + bc + ca)$

v. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

vi. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$

vii. Sum of natural numbers from 1 to n

$$\frac{n(n+1)}{2}$$

viii. Sum of squares of first n natural numbers is =

$$\frac{n(n+1)(2n+1)}{6}$$

ix. Sum of cubes of first n natural numbers is A

$$\left[\frac{n(n+1)}{2}\right]^2$$

x. HCF= (HCF of the numerators)/(LCM of the denominators)

xi. LCM= (LCM of the numerators)/HCF of the denominators

xii. Product of two numbers = Product of their H.C.F. and L.C.M

Note: When a number N is raised to any integral power m, the digit in the unit's place of the resulting value can be determined without actually evaluating the power. The digits when raised to powers will give values in which the digits in the unit's place follow a cylindrical pattern. Following is the pattern to calculate the digit in the unit's place of any derived power.

The cylindrical pattern of 0 is 0
The cylindrical pattern of 1 is 1
The cylindrical pattern of 2 is 2, 4, 8, 6
The cylindrical pattern of 3 is 3, 9, 7, 1
The cylindrical pattern of 4 is 4, 6
The cylindrical pattern of 5 is 5
The cylindrical pattern of 6 is 6
The cylindrical pattern of 7 is 7, 9, 3, 1
The cylindrical pattern of 8 is 8, 4, 2, 6
The cylindrical pattern of 9 is 9, 1

HCF models:-

If N is a composite number such that N = $a^p \cdot b^q \cdot c^r$ ….. where a, b, c are prime factors of N and p,q,r ….. are positive integers, then

(a) The number of factors of N is given by the expression (p + 1) (q + 1) (r + 1)…

(b) It can be expressed as the product of two factors in 1/2 {(p + 1) (q + 1) (r + 1)…..} ways

(c) If N is a perfect square, it can be expressed

(i)as a product of two DIFFERENT factors in 1/2 {(p + 1) (q + 1) (r + 1)….. -1} ways

(ii)as a product of two factors in 1/2 {(p + 1) (q + 1) (r + 1) ….+1} ways

(d) Sum of all factors of N = $(a^{p+1} – 1 / a – 1) . (b^{q+1} – 1 / b – 1) . (c^{r+1} – 1 / c – 1)$…..

(e) The number of co-primes of N (< N), Ø(N) = N(1 – 1/a) (1 – 1/b) (1 – 1/c) ….

(f) Sum of the numbers in (e) = N/2 . Ø(N)

(g) It can be expressed as a product of two factors in 2^{n-1}, where 'n' is the number of different prime factors of the given number N.

Exercise Questions

1. 117 * 117 + 83 * 83 = ?

a) 20698

b) 20578

c) 21698

d) 21268

2. (1/4)3 + (3/4)3 + 3(1/4)(3/4)(1/4 + 3/4) =?

a) 1/64

b)27/64

c) 49/64

d)1

3. The difference of two numbers is 1365. On dividing the larger number by the smaller, we get 6 as quotient and 15 as reminder. What is the smaller number ?

a) 240

b) 270

c) 295

d) 360

4. The 7th digit of $(202)^3$ is

a) 2
b) 4
c) 8
d) 6

5. H.C.F. of two numbers is 16. Which one of the following can never be their L.C.M

a) 32

b) 80

c) 64

d) 60

6. What is the remainder when 9 + 92 + 93 + ... + 98 is divided by 6?

a) 3
b) 2
c) 0
d)5

7. The sum of the first 100 natural numbers is divisible by

a) 2, 4 and 8

b) 2 and 4

c)2 only

d)none of these

8. For what value of 'n' will the remainder of 351^n and 352^n be the same when divided by 7?

a) 2

b)3

c)6

d)4

9. Let n be the number of different 5 digit numbers, divisible by 4 with the digits 1, 2, 3, 4, 5 and 6, no digit being repeated in the numbers. What is the value of n?

a) 144
b) 168
c)192
d)none of these

10. Find the greatest number of five digits, which is exactly divisible by 7, 10, 15, 21 and 28.

a) 99840
b) 99900
c)99960
d) 99990

Answer Key:

1.B; 2.D; 3.B; 4.C; 5.D; 6.C; 7.C; 8.B; 9.C; 10.C

Concepts and Theory

Number Theory - Tips & Tricks

1.Sum of natural numbers from 1 to n

$$\frac{n(n+1)}{2}$$

e.g Sum of natural numbers from 1 to 40 = 40(40+1)/2 = 820

2. Sum of squares of first n natural numbers is =

$$\frac{n(n+1)(2n+1)}{6}$$

3. Sum of the squares of first n even natural numbers is

$$\frac{2}{3}n(n+1)(2n+1)$$

4. Sum of cubes of first n natural numbers is

$$\left[\frac{n(n+1)}{2}\right]^2$$

5. Any number N can be represented in the decimal system of number as

$$N = n_k 10^k + n_{k-1} 10^{k-1} + n ... + n_i 10 + n_0$$

Important Formulas

i. $(a + b)(a - b) = (a^2 - b^2)$

ii. $(a + b)^2 = (a^2 + b^2 + 2ab)$

iii. $(a - b)^2 = (a^2 + b^2 - 2ab)$

iv. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

v. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

vi. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$

vii. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

viii. When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

xi. $(a + b)^2 = (a^2 + b^2 + 2ab) = (a - b)^2 + 4ab$

x. $(a - b)^2 = (a^2 + b^2 - 2ab) = (a + b)^2 - 4ab$

Some more tips:

1) $k(a + b + c) = ka + kb + kc$

2) $(a + b)(c + d) = ac + ad + bc + bd$

3) $(x + a)(x + b) = x^2 + (a + b)x + ab$

4) $(a + b)^2 - (a - b)^2 = 4ab$

5) $(a + b)^2 - (a - b)^2 = 2(a^2 + b^2)$

6) $(a + b)^3 = a^3 + b^3 + 3ab(a + b) = a^3 + 3a^2b + 3ab^2 + b^3$

7) $(a - b)^3 = a^3 - b^3 - 3ab(a - b) = a^3 - 3a^2b + 3ab^2 - b^3$

8) $1/a + 1/b = (a + b)/ab$

9) $(x + a)(x + b)(x + c) = x^3 + (a + b + c)x^2 + (ab + bc + ca)x + abc$

10) $(a + b + c)^3 = a^3 + b^3 + c^3 + 3a^2b + 3a^2c + 3b^2a + 3b^2c + 3c^2a + 3c^2b + 6abc$

Tests of Divisibility :

1. A number is divisible by 2 if it is an even number.

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4. A number is divisible by 5 if the units digit is either 5 or 0.

5.A number is divisible by 6 if the number is divisible by both 2 and 3

6. A number is divisible by 8 if the number formed by the last three digits is divisible by 8.

7. A number is divisible by 9 if the sum of the digits is divisible by 9.

8. A number is divisible by 10 if the units digit is 0.

9. A number is divisible by 11 if the difference of the sum of its digits at odd places and the sum of its digits at even places should be zero or a multiple of 11.

Some more tips:

1) A number is divisible by 12, when it is divisible by both 3 and 4.

2) A number is divisible by 25, when the last two digits are 00 or divisible by 25

3) A number is divisible by 125, if the last three digits are 000 or divisible by 125

4) A number is divisible by 27, if the sum of the digits of the number is divisible by 27.

5) A number is divisible by 125, if the number formed by last three digits is divisible by 125.

6) Number of the form 10(n-1) (where 'n' is a natural number) is always divisible by 9 if 'n' is even, such numbers are divisible by 11 also.

H.C.F and L.C.M: H.C.F stands for Highest Common Factor. The other names for H.C.F are Greatest Common Divisor (G.C.D) and Greatest Common Measure (G.C.M). The H.C.F. of two or more numbers is the greatest number that divides each one of them exactly. Two numbers are said to be co-prime if their H.C.F. is 1. The least number which is exactly divisible by each one of the given numbers is called their L.C.M. Finding L.C.M and H.C.F of Fractions

LCM= (LCM of the numerators)/(HCF of the denominators)

HCF= (HCF of the numerators)/LCM of the denominators

Product of two numbers = Product of their H.C.F. and L.C.M.

Exercise Questions

1. The difference between the local value and face value of 7 in the numeral 657903 is:

   a. 0 b. 7896 c. 6993 d. 903

2. The sum of three prime numbers is 100. If one of them exceeds another by 36, then one of the numbers is:

   a. 7 b. 29 c. 41 d. 67

3. The unit's digit in the product $(3127)^{173}$ is:

   a. 1 b. 3 c. 7 d. 9

4. (51+52+53+………+100) is equal to:

   a. 2525 b. 2975 c. 3225 d. 3775

5. 5b2 is a three-digit number with b as a missing digit. If the number is divisible by 6, the missing digit is:

   a. 2 b. 3 c. 6 d. 7

6. How many of the following numbers are divisible by 3 but not by 9?

   2133, 2343, 3474, 4131, 5286, 5340, 6336, 7347, 8115, 9276

   a. 5 b. 6 c. 7 d. None of these

7. The value of P, when 4864 x 9P2 is divisible by 12, is:

   a. 2 b. 5 c. 8 d. None of these.

8. How many of the following numbers are divisible by 132?

   264, 396, 462, 792, 968, 2178, 5184, 6336

   a. 4 b. 5 c. 6 d. 7

9. The number 311311311311311311311 is:

a. divisible by 3 but not by 11 b. divisible by 11 but not by3 c. divisible by both 3 and 11 d. neither divisible by 3 nor by 11.

10. The largest natural number which exactly divides the product of any four consecutive natural numbers is:

a. 6 b. 12 c. 24 d. 120

11. The sum of three consecutive odd numbers is always divisible by:

I. 2 II. 3 III. 5 IV. 6

a. Only I b. Only II c. Only I and II d. Only II and IV

12. The least number which must be subtracted from 6709 to make it exactly divisible by 9 is:

a. 2 b. 3 c. 4 d. 5

13. The least number by which 72 must be multiplied in order to produce a multiple of 112, is:

a. 6 b. 12 c. 14 d. 18

14. On dividing a number by 999, the quotient is 366 and the remainder is 103. The number is:

a. 364724 b. 365387 c. 365737 d. 366757

15. When a number is divided by 31, the remainder is 29. When the same number is divided by 16, what will be the remainder?

a. 11 b. 13 c. 15 d. Data inadequate

16. A number when divided by 6 leaves a remainder 3. When the square of the same number is divided by 6, the remainder is:

a. 0 b. 1 c. 2 d. 3

17. If x is a whole number, then $x^2(x^2-1)$ is always divisible by:

a. 12 b. 24 c. 12-x d. multiple of 12

18. A number when divided successively in order by 4, 5 and 6. The remainders were respectively 2, 3 and 4. The number is:

a. 214 b. 476 c. 954 d. 1908

19. A number when divided by 3 leaves a remainder 1. When the quotient is divided by 2, it leaves a remainder 1. What will be the remainder when the number is divided by 6?

    a. 2 b. 3 c. 4 d. 5

20. A number when divided b the sum of 555 and 445 gives two times their difference as quotient and 30 as the remainder. The number is:

    a. 1220 b. 1250 c. 2 2030 d. 22030

Answer & Explanations

1. Ans: c.

    (Local value)-(Face value)= (7000-7)=6993.

2. Ans: d

    x+(x+36)+y=100 => 2x+y=64

    Therefore y must be even prime, which is 2.

    Therefore 2x+2=64 => x=31

    Third prime number= (x+36)= (31+36)= 67.

3. Ans: c

    Unit digit in $(3127)^{173}$ = Unit digit in 7^{173}. Now, 7^4 gives unit digit 1.

    Therefore, $7^{173} = (7^4)^{43} * 7^1$. Thus, 7^{173} gives unit digit 7.

4. Ans: d

    (51+52+53+………+100) = (1+2+3+…….+100)- (1+2+3+4+……+50)

    = (100*101)/2 - (50*51)/2

    = (5050-1275)= 3775.

5. Ans: a

    Let the number be 5b2. Clearly, it is divisible by 2.

    Now, 5+b+2= (7+b) must be divisible by 3. So, b= 2.

6. Ans:b

   Taking the sum of the digits, we have:

   S1=9, S2=12, S3=18, S4= 9, S5=21, S6= 12, S7=18, S8=21, S9=15, S10=24.

   Clearly S2, S5, S6, S8, S9, S10 are all divisible by 3 but not by 9. So, the number of required numbers= 6.

7. Ans: d

   Since 4864 is divisible by 4, so 9P2 must be divisible by 3.

   Therefore (11+P) must be divisible by 3.

   Therefore least value of P is 1

8. Ans:a.

   A number is divisible by 132, if it is divisible by each one of 11, 3 and 4.

   Clearly, 968 is not divisible by 3. None of 462 and 2178 is divisible by 4.

   Also, 5184 is not divisible by 11.

   Each one of remaining 4 is divisible by each one of 11, 3 and 4 and therefore, by 132.

9. Ans: d

   Sum of digits= 35 and so it is not divisible by 3.

   (Sum of digits at odd places)- (Sum of digits at even places)= 19-16=3, not divisible by 11.

   So, the given number is neither divided by 3 nor by 11.

10. Ans: c

    Required number= 1*2*3*4=24

11. Ans: b

    Let the three consecutive odd numbers be (2x+1), (2x+3) and (2x+5). Their sum = (6x=9)= 3 (2x+3), which is always divisible by 3.

12. Ans: c.

    On dividing 6709 by 9, we get remainder= 4

Therefore, required number to be subtracted= 4

13. Ans: c

    Required number is divisible by 72 as well as by 112, if it is divisible by their LCM, which is 1008.

    Now, 1008 when divided by 72, gives quotient= 14.

    Therefore, required number= 14.

14. Ans: c.

    Required number= 999*366+103= (1000-1)*366+103= 366000-366+103= 365737.

15. Ans: d

    Number= (31 * Q)+ 29. Given data is inadequate.

16. Ans: d

    Let x=6q+3. Then, x^2= $(6q+3)^2$= $36q^2$+36q+9= $6(6q^2+6q+1)+3$.

    So, when 2n is divided by 4, remainder =3.

17. Ans: a

    Putting x=2, we get $2^2(2^2-1)$= 12. So, $x^2(x^2-1)$ is always divisible by 12.

18. Ans: a

4	x
5	y - 2
6	z - 3
	1 - 4

Z= 6*1 +4= 10.

Y= 5z+3= 53

X= 4y+2= 214.

19. Ans: c

    Let n=3q+1 and let q= 2p+1. Then, n= 3(2p+1)+1= 6p+4

Therefore, the number when divided by 6, we get remainder= 4

20. Ans: d

Required number = (555+445)*2*110+30 = 220000+30= 220030.