Probability- Key Points

Probability of an event= (No of occurrences of an event)/(Total no. of possible outcomes)

P(E) = n(E)/n(U)
where n(E)= no. of occurrences of event; n(U)= total possible outcomes

Probability of head in a toss=1/2
Probability of tails in a toss=1/2
Probability of any side in a throw of dice=1/6
Probability of getting a sign in a deck of cards= 1/4
Probability of getting an ace; king; queen etc= 1/13
Probability of occurrence of 2 independent events= P(A)*P(B)

Exercise Questions

1. If 10 letters are to be placed in 10 addressed envelopes, then what is the probability that at least one letter is placed in wrong addressed envelope?
a.1/10!
b.1/9!
c.1- (1/10!)
d.9/10

2.Two letters are selected randomly from English alphabet simultaneously. What is the probability that one is a constant and the other a vowel?
a.10/67
b.21/65
c.44/65
d.2/13

3. Two dice are rolled together. The probability that the sum of the numbers shown on the two dice is maximum or minimum is……
a.1/18
b.1/9
c.1/6
d.1/36

4. A bag has 5 red balls and 3 green balls and another bag contains 4 green balls and 3 red balls. If one ball is drawn from each bag, then what is the probability that both are green balls?
a.5/14
b.3/14
c.11/14
d.9/14

5. Two cards are drawn at random from a pack of cards. The probability that both are queens or diamonds is……
a.20/221

b.15/221
c.13/221
d.14/221

6. If a coin is tossed 6 times, then the probability of getting exactly 1 head is ………..
a.1/32
b.13/32
c.5/32
d.3/32

7. Amit throws a biased coin on which the head appears in 65% of the situations. In a game involving this coin, if Amit is paid Rs.15 per head and he has to pay Rs.20 for a tail, then in the long run, per game Amit makes an average……
a.Profit of Rs.2.25
b.loss of Rs.2.25
c.profit of Rs.2.75
d.loss of Rs.2.75

8. From a box containing a dozen bulbs, of which exactly one half are good, and four bulbs are chosen at random to fit into the four bulb holders in a room. The probability that the room gets lighted is…………
a.2/3
b.1/3
c.33/44
d. 32/33

9. From a bag containing 6 pink and 8 orange balls, 8 balls are drawn at random. The probability that 5 of them are pink and the rest are orange is…………
a.16/143
b.19/143
c.17/143
d. 13/143

10. A bag contains 5 five-rupee coins, 8 two-rupee coins and 7 one-rupee coins. If four coins are drawn from the bag at random, then find the odds in favour of the draw yielding the maximum possible amount.
a.1:968
b.968:969
c.1:969
d.969:968

Answer Key

1.c; 2.b; 3.a; 4.b; 5.d; 6.d; 7.c; 8.d; 9.a; 10.a

Concepts and Theory

1. A probability of 0 means that the outcome cannot happen. A probability of 1 means that the outcome will definitely happen. And in between 0 and 1 means that the outcome may happen.

Example with a coin

When a coin is tossed the outcome (or event) can be heads or tails. What is the probability it is tails? Since each outcome, heads or tails, is equally likely we can say that the probability of each is 0.5.
p(coin toss is tails)= 1/2

Basic rule of probability

More generally we can say that where there are n equally likely outcomes then the probability of each of these possibilities will be 1/n. So we can say that
p(outcome)= (number of ways it can happen)/(total number of possible outcomes)

This is the basis of all probability questions in the GMAT

Example with die

What is the probability of rolling a 6 when you throw a 6 sided die.Each number from 1 to 6 is equally likely to be thrown and only one of those outcomes is a 6 so using the general rule we can say that
p(throw a six)= 1/6

Example with cards

If you pick a card at random from a deck of cards what is the probability that it is an ace?
There are 52 cards in a pack and those there are 4 aces so
p (an ace)= 4/52 - 1/13

The probability two outcomes for independent events both occur can be found by multiplying their probabilities.
p(A and B)- p(A)*p(B)

Example with coins

What is the probability of throwing two heads in a row when tossing a coin?
This is the same as asking what the probability that the first coin tossed will be head AND the second coin tossed will be a head.
So the probability that of tossing two heads in a row is 1/4

Example with a jar

A jar contains 2 red balls and 4 green balls. What is the probability that two balls selected at random from the jar are both green?
Each ball is equally likely to be selected from the jar so we can work out the probability of the first ball

selected being green.
Here is where we need to be careful, once we have taken 1 green ball out of the jar, the jar contains only 3 green balls ans 2 red balls so Now we can say that,So the probability that of picking out two green balls is 2/5.

Total Probability Formula.

P(A) = P(AjH1)P(H1) + ¢ ¢ ¢ + P(AjHn)P(Hn):

Total probability
Events H1;H2; : : : ;Hn form a partition of the sample space S if
(i) They are mutually exclusive (Hi ¢ Hj = ;; i 6= j) and
(ii) Their union is the sample space S; i=1 Hi=S;
The events H1; : : : ;Hn are usually called hypotheses and from their definition follows that P(H1) +¢ ¢ ¢ + P(Hn) = 1 (= P(S)):

Let the event of interest A happens under any of the hypotheses Hi with a known (conditional) probability P(AjHi): Assume, in addition, that the probabilities of hypotheses H1; : : : ;Hn are known. Then P(A) can be calculated using the total probability formula.

Total Probability Formula:P(A) = P(AjH1)P(H1) + ¢ ¢ ¢ + P(AjHn)P(Hn):
The probability of A is the weighted average of the conditional probabilities P(AjHi) with weights P(Hi):

Bayes Formula:

Let the event of interest A happens under any of hypothesesHi with a known (conditional) probability P(AjHi): Assume, in addition, that the probabilities of hypotheses H1; : : : ;Hn are known (prior probabilities). Then the conditional (posterior) probability of the hypothesis Hi; i = 1; 2; : : : ; n, given that event A happened, is
P(HijA) = P(AjHi)P(Hi) P(A) ; where P(A) = P(AjH1)P(H1) + ¢ ¢ ¢ + P(AjHn)P(Hn):

Assume that out of N coins in a box, one has heads at both sides. Such “two-headed” coin can be purchased in Spencer stores. Assume that a coin is selected at random from the box, and without inspecting it, flipped k times. All k times the coin landed up heads. What is the probability that two headed coin was selected?

Denote with Ak the event that randomly selected coin lands heads up k times. The hypotheses areH1- the coin is two headed, and H2 the coin is fair. It is easy to see that P(H1) = 1=N and P(H2) = (N ¡ 1)=N. The conditional probabilities are P(AkjH1) = 1 for any k, and P(AkjH2) = 1=2k:
By total probability formula,
P(Ak) = (2k + N- 1)/2kN and

P(H1jAk) = 2k/(2k + N- 1)

Conditional Probability

Problem: A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test:

Analysis: This problem describes a conditional probability since it asks us to find the probability that the second test was passed given that the first test was passed. In the last lesson, the notation for conditional probability was used in the statement of Multiplication Rule 2.

Multiplication Rule 2:
When two events, A and B, are dependent, the probability of both occurring is P(A and B)= P(A).P(B|A)The formula for the Conditional probability of an event can be derived from Multiplication on Rule 2 as follows.

Step1: P(A and B)=P(A).P(B|A) start with multiplication rule 2.
Step2: P(A and B)/P(A) = (P(A).P(B|A))/P(A) Divide both sides of the equation by P(A)
Step3: P(A and B)/P(A) = P(B|A) Cancel P(A)s on right-hand side of the equation
Step4: P(A and B)/P(A) = We have derived the formula for conditional probability

Problem: A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?
Solution: P(Second|First) = P(First and Second)/P(First)= = 0.25/0.42 = 0.60 = 60%

Exercise Questions

1. If a number is chosen at random from the set {1, 2, 3…, 100}, then the probability that the chosen number is a perfect cube is

a. 1/25 b. 1/2 c. 4/13 d. 1/10

2. What is the probability of getting at least one six in a single throw of three unbiased dice?

a. 1/6 b. 125/216 c. 1/36 d. 81/216 e. 91/216

3. In a simultaneous throw of two dice, what is the probability of getting a doublet?

a. 1/6 b. 1/4 c. 2/3 d. 3/7

4. A bag contains 4 red balls, 5 green balls and 6 white balls. A ball is drawn at random from the box. What is the probability that the ball drawn is either red or green?

a. 2/5 b. 3/5 c. 1/5 d. 7/15

5. When 4 dice are thrown, what is the probability that the same number appears on each of them?

a. 1/36 b. 1/18 c. 1/216 d. 1/5

6. The probability that it is Friday and that a student is absent is 0.03. Since there are 5 school days in a week, the probability that it is Friday is 0.2. What is the probability that a student is absent given that today is Friday?

a. 10% b. 15% c. 12% d. 13%

7. Two dice are rolled. The probability of getting a sum of at least 9 is

a. 13/36 b. 5/18 c. 35/36 d. 11/36

8. If four cards are drawn at random from a well shuffled pack of cards, what is the probability that each card is an ace?

a. $6/{}^{52}C_4$ b. $4/{}^{52}C_4$ c. $1/{}^{52}C_4$ d. $3/{}^{52}C_4$

9. A person tosses an unbiased coin. When head turns up, he gets Rs.8 and tail turns up he loses Rs.4. If 3 coins are tossed, what is probability that the gets a profit of Rs.12?

a. 3/8 b. 5/8 c. 3/4 d. 1/8

10. A number n is chosen from {2, 4, 6 … 48}. The probability that 'n' satisfies the equation (2x – 6) (3x + 12) (x – 6) (x – 10) = 0 is

a. 1/24 b. 1/12 c. 1/8 d. 1/6

Directions for questions: 11 to 13: These questions are based on the following data.

A box contains 12 mangoes out of which 4 are spoilt. If four mangoes are chosen at random, find the probability that

11. All the four mangoes are spoiled.

a. 1/495 b. 494/495 c. 1/395 d. 394/395

12. Not all the mangoes are spoiled.

a. 1/495 b. 394/395 c. 494/495 d. 1/395

13. Exactly three are not spoiled.

a. 116/495 b. 224/495 c. 129/495 d. 187/495

14. A number is selected at random from first thirty natural numbers. What is the chance that it is a multiple of either 3 or 13?

a.17/30 b. 2/5 c. 11/30 d. 4/15

Directions for questions: 15 to 17: These questions are based on the following data.

If the numbers 1 to 100 are written on 100 pieces of paper, (one on each) and one piece is picked at random, then

15. What is the probability that the number drawn is neither prime nor composite?

a. 1/50 b. 1/25 c. 1/100 d. 1

16. Find the probability that the number drawn is a multiple of 6 and 8.

a. 3/50 b. 2/25 c. 1/50 d. 1/25

17. Find the probability that the number drawn is a factor of 50.

a. 1/25 b. 1/50 c. 3/25 d. 3/50

18. Out of 7 fruits in a basket, 2 are rotten. If two fruits are drawn at random from the basket, the probability of both being rotten is

a. 1/21 b. 10/21 c. 20/21 d. 11/21

19. The probability that a number selected at random from first 50 natural numbers is a composite number is

a. 21/25 b. 17/25 c. 4/25 d. 8/25

20. If six persons sit around a table, the probability that some specified three of them are always together is

a. 1/20 b. 3/10 c. 1/5 d. 4/5

Answer & Explanations

1. Exp: We have 1, 8, 27 and 64 as perfect cubes from 1 to 100. Thus, the probability of picking a perfect cube is 4/100 = 1/25

2. Exp: Find the number of cases in which none of the digits show a '6'.

i.e. all three dice show a number other than '6', 5 * 5 *5 = 125 cases.

Total possible outcomes when three dice are thrown = 216.

The number of outcomes in which at least one die shows a '6' = Total possible outcomes when three dice are thrown - Number of outcomes in which none of them show '6'.

= 216 - 125 = 91.

The required probability = 91/216.

3. Exp: In a simultaneous throw of two dice, n(S) = (6 x 6)= 36.

Let E = event of getting a doublet = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}.

Therefore, P(E) = n (E)/ n(S) = 6/36 = 1/6.

4. Exp: Total number of balls = (4 + 5 + 6) = 15.

Therefore, n(S) = 15.

Let E_1 = event of drawing a red ball.

and E_2 = event of drawing a green ball.

Then, E_1 n E_2 = f.

P (E_1 n E_2) = P(E_1) + P(E_2) = (4/15 + 5/15) = 9/15 = 3/5.

5. Exp: Sample space (Denominator): When 4 dice are thrown simultaneously, then the total number of possible outcomes is 64 = 1296.

Event (Numerator): The chances that all the dice show same number (1,1,1,1), (2,2,2,2), (3,3,3,3), (4,4,4,4), (5,5,5,5), (6,6,6,6)} is 6.

Probability = Event/ Sample space = 6/64 = 1/63 = 1/216.

6. Exp: P(Absent|Friday) = $\frac{\text{P(Friday and abscent)}}{\text{P(Friday)}}$ = 0.03/0.2 = 15%

7. Exp: Sum of 9 = {(3,6) (6,3)} = 2ways.

Sum of 10 = {(5,5) (6,4) (4,6) (5,5)} = 4 ways.

Sum of 11 = {(6,5) (5,6)} = 2 ways.

Sum of 12 = {(6,6) (6,6)} = 2 ways.

Therefore, Favourable cases = 10

Total cases = 6 x 6 = 36.

Therefore, Probability = 10/36 = 5/18.

8. Exp: Four cards can be drawn from a pack in ${}^{52}C_4$ ways.

Let E be the event of each card being an ace.

This can be done in ${}^{4}C_4$ i.e, 1 way.

So P(E) = $1/{}^{52}C_4$

9. Exp: When a person tosses two heads and one tail, he will get Rs.12. When three coins are tossed, total outcomes = 2^3 = 8. Favourable out comes i.e, two heads and one tail is = {HHT, HTH, THH} = 3ways. Therefore, required probability = 3/8.

10. Exp: Given, set is {2, 4, 6 … 48}

n(s) = 24

The roots of given equations are 3,4,4,10. The number of chosen from the set are 4, 10, which are the roots of given equation.

n(E) = 2

Therefore, required probability = 2/24 = 1/12.

11. Exp: Out of 12,8 are good and 4 are spoiled.

Required probability = ${}^{4}C_{4}/{}^{12}C_{4}$ = 1/495.

12. Exp: Required probability = 1 – 1/495 = 494/495.

13. Exp: Required probability = $\frac{{}^{8}C_{3} \,.\, {}^{4}C_{1}}{{}^{12}C_{4}} = \frac{56 \times 4}{495}$ = 224/495.

14. Exp: The probability that the number is a multiple of 3 is 10/30. (Since 3*10 = 30).

Similarly the probability that the number is a multiple of 13 is 2/30. {Since 13*2 = 26).

Neither 3 nor 13 has common multiple from 1 to 30. Hence these events are mutually exclusive events. Therefore chance that the selected number is a multiple of 3 or 13 is (10+2)/30 = 2/5.

15. Exp: There are 25 primes, 74 composite numbers from 1 to 100. The number which is neither prime nor composite is 1.

Therefore, required probability = 1/100.

16. Exp: From 1 to 100 there are 4 numbers which are multiples of 6 and 8. (i.e, multiples of 24)

Therefore, required probability = 4 /100 = 1/25

17. Exp: The factors of 50 are 1, 2, 5, 10, 25, 50

Therefore, required probability = 6/100 = 3/50.

18. Exp: The number of exhaustive events = ${}^{7}C_{2}$ = 21.

Let E be event of the 2 fruits being rotten. The number of favourable cases are $^2C_2$ = 1 way. \ Required probability = 1/21.

19. Exp: The number of exhaustive events = $^{50}C_1$ = 50. We have 15 primes from 1 to 50.

Number of favourable cases are 34.

Therefore, Required probability = 34/50 = 17/25.

20. Exp: There are six persons and three of them are grouped together. Since it is a circle, this can be done in 3! 3! ways. Therefore, P = 3!3!/5 = 3/10