WBJEE - 2016

MATHEMATICS

Q.No.	✚	□	✿	❍
01	A	C	B	B
02	B	B	A	B
03	C	A	C	C
04	A	B	C	C
05	A	A	B	C
06	B	C	B	C
07	B	C	A	D
08	C	C	C	A
09	D	D	C	C
10	A	C	A	B
11	B	C	B	A
12	A	C	A	B
13	D	A	A	A
14	B	B	D	C
15	B	C	C	C
16	C	A	B	B
17	C	A	A	B
18	C	B	B	A
19	C	B	A	C
20	D	C	C	C
21	A	D	C	A
22	C	A	C	B
23	B	B	D	A
24	A	A	C	A
25	B	D	C	D
26	A	B	C	C
27	C	B	A	B
28	C	C	B	A
29	B	C	C	B
30	B	C	A	A
31	A	C	A	C
32	C	D	B	C
33	C	A	B	C
34	A	C	C	D
35	B	B	D	C
36	A	A	A	C
37	A	B	B	C
38	D	A	A	A
39	C	C	D	B
40	B	C	B	C
41	A	B	B	A
42	B	B	C	A
43	A	A	C	B
44	C	C	C	B
45	C	C	C	C
46	C	A	D	D
47	D	B	A	A
48	C	A	C	B
49	C	A	B	A
50	C	D	A	D
51	B	A	A	D
52	D	C	A	B
53	B	B	A	A
54	C	B	C	A
55	D	B	B	D
56	B	D	B	A
57	A	B	B	A
58	A	C	D	A
59	D	D	B	C
60	A	B	C	B
61	A	A	D	B
62	A	A	B	B
63	C	D	A	D
64	B	A	A	B
65	B	A	D	C
66	B,D	A,B	B,D	A,B
67	B,D	A,C	A,B	B,C
68	A,B	B,D	A,C	B,D
69	B,C	B,D	A,C	A,B
70	B,D	A,B	A,B	A,C
71	A,B	B,C	A,C	A,C
72	A,C	B,D	B,D	A,B
73	A,C	A,B	B,D	A,C
74	A,B	A,C	A,B	B,D
75	A,C	A,C	B,C	B,D

Code-✚

ANSWERS & HINT
for
WBJEE - 2016
SUB : MATHEMATICS

CATEGORY - I (Q1 to Q50)

Only one answer is correct. Correct answer will fetch full marks 1. Incorrect answer or any combination of more than one answer will fetch – ¼ marks.

1. Let A and B two events such that $P\left(A\cap B\right)=\frac{1}{6}, P(A\cup B)=\frac{31}{45}$ and $P(\bar{B})=\frac{7}{10}$ then

(A) A and B are independent

(B) A and B are mutually exclusive

(C) $P\left(\frac{A}{B}\right)<\frac{1}{6}$

(D) $P\left(\frac{B}{A}\right)<\frac{1}{6}$

Ans : (A)

Hint : $P\left(\bar{B}\right)=\frac{7}{10}\Rightarrow P(B)=\frac{3}{10}$

$$P(A\bigcup B)=P(A)+P(B)-P(A\cap B)\Rightarrow P(A)=\frac{5}{9}$$

$$\therefore P(A)\times P(B)=\frac{5}{9}\times\frac{3}{10}=\frac{1}{6}=P(A\bigcap B)$$

$\Rightarrow$ A, B are independent

2. The value of cos 15° $\cos 7\frac{1^\circ}{2}\sin 7\frac{1^\circ}{2}$ is

(A) $\frac{1}{2}$

(B) $\frac{1}{8}$

(C) $\frac{1}{4}$

(D) $\frac{1}{16}$

Ans : (B)

Hint : $\cos 15^0\cos 7\frac{1}{2}^0\sin 7\frac{1}{2}^0=\frac{\left(2\sin 7\frac{1}{2}^0\cos 7\frac{1}{2}^0\right)\cos 15^0}{2}=\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$

3. The smallest positive root of the equation tan x – x = 0 lies in

(A) $(0, \pi/2)$

(B) $(\pi/2, \pi)$

(C) $\left(\pi,\frac{3\pi}{2}\right)$

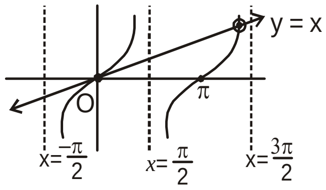
(D) $\left(\frac{3\pi}{2},2\pi\right)$

Ans : (C)

Hint : $\tan x - x = 0 \Rightarrow \tan x = x$

Solutions are abscisse of points of intersection of the curves $y = \tan x$ and $y = x$.

It is clearly visible that solution lies in $\left(\pi, \frac{3\pi}{2}\right)$.



4. If in a triangle ABC,AD, BE and CF are the altitudes and R is the circumradius, then the radius of the circumcircle of ΔDEF is

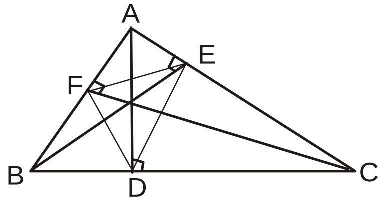
(A) $\frac{R}{2}$ (B) $\frac{2R}{3}$ (C) $\frac{1}{3}R$ (D) None of these

Ans : (A)

Hint : Let, circumradius of ΔDEF be R′. We know, $\angle FDE = 180° - 2A$ and $FE = R\sin 2A$

Now, by sine rule in ΔDEF,

$$2R' = \frac{EF}{\sin\angle FDE} = \frac{R\sin 2A}{\sin(180° - 2A)} \Rightarrow R' = \frac{R}{2}$$



5. The points $(-a, -b)$, (a, b), $(0, 0)$ and (a^2, ab), $a \neq 0$, $b \neq 0$ are always lie on this line.

Hence, collinear

(A) collinear (B) vertices of a parallelogram

(C) vertices of a rectangle (D) lie on a circle

Ans : (A)

Hint : The straight line through (a, b) and $(-a, -b)$ is $bx = ay$. Obviously, $(0, 0)$ and (a^2, ab) always lie on this line

6. The line AB cuts off equal intercepts 2a from the axes. From any point P on the line AB perpendiculars PR and PS are drawn on the axes. Locus of mid-point of RS is

(A) $x - y = \frac{a}{2}$ (B) $x + y = a$

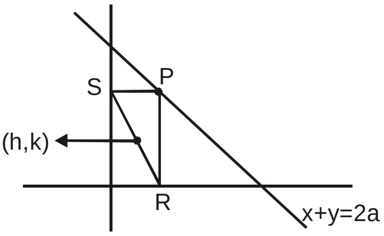
(C) $x^2 + y^2 = 4a^2$ (D) $x^2 - y^2 = 2a^2$

Ans : (B)

Hint : Equation of AB is $x + y = 2a$

Let, co-ordinates of the mid-point be (h, k). So, R and S are $(2h, 0)$ and $(0, 2k)$. Therefore, P must be $(2h, 2k)$.

Now P lies on ΔB.



$\therefore 2h + 2k = 2a$

$\Rightarrow x + y = a$

7. $x + 8y - 22 = 0$, $5x + 2y - 34 = 0$, $2x - 3y + 13 = 0$ are the three sides of a triangle. The area of the triangle is

(A) 36 square unit
(B) 19 square unit
(C) 42 square unit
(D) 72 square unit

Ans : (B)

Hint : If AB denotes : $x+8y-22 = 0 \longrightarrow (1)$
BC denotes : $5x+2y-34 = 0 \longrightarrow (2)$
and CA denotes : $2x-3y+13 =0 \longrightarrow (3)$

Then solving equations (1), (2) and (3), we get

$A \equiv (-2,3)$, $B \equiv (6,2)$ and $C \equiv (4,7)$.

Hence, area of ΔABc is 19 square units

8. The line through the points (a, b) and (–a, –b) passes through the point

(A) (1, 1)
(B) (3a, –2b)
(C) (a^2, ab)
(D) (a, b)

Ans : (C) **

**** Note : The point in Option D is already in the question.**

Hint : The line through (a, b) and (–a, –b) has the equation bx = ay. Hence, (a^2, ab) is always on the line.

9. The locus of the point of intersection of the straight lines $\frac{x}{a}+\frac{y}{b}=K$ and $\frac{x}{a}-\frac{y}{b}=\frac{1}{k}$, where k is a non-zero real variable, is given by

(A) a straight line
(B) an ellipse
(C) a parabola
(D) a hyperbola

Ans : (D)

Hint : Let the point intersection be (α, β).

so, $\frac{\alpha}{a}+\frac{\beta}{b}=k$ and $\frac{\alpha}{a}-\frac{\beta}{b}=\frac{1}{k}$

$$\Rightarrow \frac{\alpha^2}{a^2}-\frac{\beta^2}{b^2}=1$$

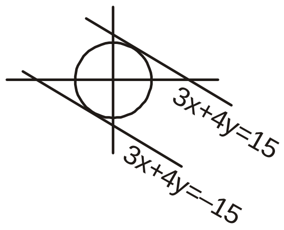
$\therefore$ Locus : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ which is equation of a hyperbola.

10. The equation of a line parallel to the line $3x + 4y = 0$ and touching the circle $x^2 + y^2 = 9$ in the first quadrant is

(A) $3x + 4y = 15$
(B) $3x + 4y = 45$
(C) $3x + 4y = 9$
(D) $3x + 4y = 27$

Ans : (A)

Hint : Let, the equation be $3x + 4y = k$



then, $y=-\frac{3}{4}x+\frac{k}{4}$. By condition of tangency $\left(\frac{k}{4}\right)^2=9\left(1+\left(\frac{-3}{4}\right)^2\right) \Rightarrow k=\pm 15$

$3x + 4y = 15$ touches in the first quadrant.

11. A line passing through the point of intersection of x+y=4 and x–y=2 makes an angle $\tan^{-1}\left(\frac{3}{4}\right)$ with the x-axis. It intersects the parabola $y^2=4(x-3)$ at points (x_1, y_1) and (x_2, y_2) respectively. Then $|x_1-x_2|$ is equal to

(A) $\frac{16}{9}$ (B) $\frac{32}{9}$ (C) $\frac{40}{9}$ (D) $\frac{80}{9}$

Ans : (B)

Hint : Point of intersection of x+y=4 and x–y=2 is ≡ (3, 1)

The line though this making an angle $\tan^{-1}\frac{3}{4}$ with the x–axis

is $(y-1)=\frac{3}{4}(x-3)$

$\Rightarrow y=\frac{3x}{4}-\frac{5}{4}=\frac{3x-5}{4}$

Putting y in $y^2=4(x-3)$, we have

$9x^2-94x+217=0$

$\Rightarrow x_1+x_2=\frac{94}{9}$ and $x_1 x_2=\frac{217}{9}$

$\Rightarrow |x_1-x_2|=\sqrt{(x_1+x_2)^2-4x_1x_2} = \frac{32}{9}$

12. Then equation of auxiliary circle of the ellipse $16x^2+25y^2+32x-100y=284$ is

(A) $x^2+y^2+2x-4y-20=0$ (B) $x^2+y^2+2x-4y=0$

(C) $(x+1)^2+(y-2)^2=400$ (D) $(x+1)^2+(y-2)^2=225$

Ans : (A)

Hint : Simplifying the given equation, we have the ellipse as : $\frac{(x+1)^2}{25}+\frac{(y-2)^2}{16}=1$

So, the auxilliary circle is $(x+1)^2+(y-2)^2=25 \quad \Rightarrow x^2+y^2+2x-4y-20=0$

13. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that ΔOPQ is equilateral, O being the centre. Then the eccentricity e satisfies

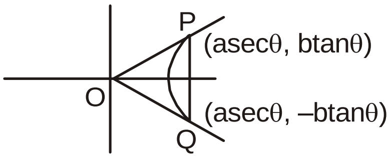
(A) $1<e<\frac{2}{\sqrt{3}}$ (B) $e=\frac{2}{\sqrt{2}}$ (C) $e=\frac{\sqrt{3}}{2}$ (D) $e>\frac{2}{\sqrt{3}}$

Ans : (D)

Hint : ∵ ΔOPQ is equilateral,

OP = PQ

$\Rightarrow a^2\sec^2\theta+b^2\tan^2\theta=(2b\tan\theta)^2$



$\Rightarrow a^2 \sec^2\theta = 3b^2 \tan^2\theta$

$\Rightarrow \sin^2\theta = \dfrac{a^2}{3b^2}$

Now, $\sin^2\theta < 1$

$\Rightarrow \dfrac{a^2}{3b^2} < 1$

$\Rightarrow \dfrac{b^2}{a^2} > \dfrac{1}{3} \quad \Rightarrow 1 + \dfrac{b^2}{a^2} > \dfrac{4}{3} \quad \Rightarrow e^2 > \dfrac{4}{3} \quad \Rightarrow e > \dfrac{2}{\sqrt{3}}$

14. If the vertex of the conic $y^2 - 4y = 4x - 4a$ always lies between the straight lines; x+y=3 and 2x+2y–1=0 then

(A) 2<a<4 (B) $-\dfrac{1}{2}$<a<2 (C) 0<a<2 (D) $-\dfrac{1}{2} < a < \dfrac{3}{2}$

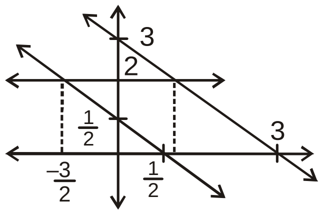
Ans : (B)

Hint : $y^2 - 4y + 4 = 4x - 4a + 4 \Rightarrow (y-2)^2 = 4(x-(a-1))$

$\therefore$ vertex : (a –1, 2)

Clearly, $\dfrac{-3}{2} < a - 1 < 1$

$\Rightarrow \dfrac{-1}{2} < a < 2$



15. A straight line joining the points (1,1,1) and (0,0,0) intersects the plane 2x+2y+z=10 at

(A) (1, 2, 5) (B) (2, 2, 2) (C) (2, 1, 5) (D) (1, 1, 6)

Ans : (B)

Hint : D.R. of line (1,1,1)

$\therefore$ let point be (k, k, k)

$\therefore$ 2k+2k+k = 10 $\Rightarrow$ 5k = 10 $\Rightarrow$ k = 2

Hence point : (2,2,2)

16. Angle between the planes x+y+2z=6 and 2x–y+z=9 is

(A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{6}$ (C) $\dfrac{\pi}{3}$ (D) $\dfrac{\pi}{2}$

Ans : (C)

Hint : x+y+2z = 6; 2x –y + z = 9

$\therefore$ Angle between the planes = angle between the normals :

$$\theta = \cos^{-1}\left(\frac{1\times 2 + 1(-1) + 2\times 1}{\sqrt{1^2+1^2+2^2}\,.\,\sqrt{2^2+(-1)^2+1^2}}\right)$$

$$= \cos^{-1}\left(\frac{4-1}{6}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

17. If $y=(1+x)(1+x^2)(1+x^4).....(1+x^{2n})$ then the value of $\left(\frac{dy}{dx}\right)$ at x = 0 is

(A) 0 (B) –1 (C) 1 (D) 2

Ans : (C)

Hint : $y=(1+x)(1+x^2)(1+x^4).....(1+x^{2n})$

$$\Rightarrow \ln y=\ln(1+x)+\ln(1+x^2)...+\ln(1+x^{2n})$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\frac{1}{1+x}+\frac{2x}{1+x^2}+\frac{3x^2}{1+x^3}+.....+\frac{2nx^{2n-1}}{(1+x^{2n})}$$

$$\Rightarrow \frac{dy}{dx}=y.\left(\frac{1}{1+x}+\frac{2x}{1+x^2}+\frac{3x^2}{1+x^3}+.....+\frac{2nx^{2n-1}}{1+x^{2n}}\right)$$

$$\Rightarrow \left|\frac{dy}{dx}\right|_{x=0}=1.1=1$$

18. If f(x) is an odd differentiable function defined on $(-\infty,\infty)$ such that f′(3) = 2, then f′(–3) equal to

(A) 0 (B) 1 (C) 2 (D) 4

Ans : (C)

Hint : Let f(x) = –f(–x)

⇒ f′(x) = –f′(–x). (–1) = f′(–x)

∴ f′ (–3) = f′ (3) = 2

19. $\lim\limits_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\frac{(1-\sqrt{x})}{(1-x)}}$

(A) is 1 (B) does not exist (C) is $\sqrt{\frac{2}{3}}$ (D) is/n 2

Ans : (C)

Hint : $\lim\limits_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}}=\lim\limits_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\frac{1}{1+\sqrt{x}}}=\left(\frac{1+1}{2+1}\right)^{\frac{1}{1+1}}=\left(\frac{2}{3}\right)^{\frac{1}{2}}$

20. If $f(x)=\tan^{-1}\left[\frac{\log\left(\frac{e}{x^2}\right)}{\log(ex^2)}\right]+\tan^{-1}\left[\frac{3+2\log x}{1-6\log x}\right]$ then the value of f″(x) is

(A) x^2 (B) x (C) 1 (D) 0

Ans : (D)

Hint : $f(x)=\tan^{-1}\left(\frac{1-2\log x}{1+2\log x}\right)+\tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$

let, $2\log x = \tan\theta$

$3 = \tan\alpha$

$\therefore\ f(x) = \frac{\pi}{4} - \cancel{\theta} + \alpha + \cancel{\theta}$

$= \frac{\pi}{4} + \tan^{-1}(3) = \text{constant}$

$\therefore f''(x) = 0$

21. $\int \frac{\log\sqrt{x}}{3x}dx$ is equal to

(A) $\frac{1}{3}\left(\log\sqrt{x}\right)^2 + c$

(B) $\frac{2}{3}\left(\log\sqrt{x}\right)^2 + c$

(C) $\frac{2}{3}(\log x)^2 + c$

(D) $\frac{1}{3}(\log x)^2 + c$

Ans : (A)

Hint : $\int \frac{\log\sqrt{x}}{3x}\, dx = I$

Let $\log\sqrt{x} = z \Rightarrow \frac{1}{2x}dx = dz$

$\therefore I = \int \frac{2z}{3}dz \quad = \frac{2}{3}\int z dz$

$= \frac{\cancel{2}}{3}\cdot\frac{z^2}{\cancel{2}} + c$

$= \frac{1}{3}.\left(\log\sqrt{x}\right)^2 + c$

22. $\int 2^x\left(f'(x) + f(x)\log 2\right)dx$ is equal to

(A) $2^x f'(x)+c$

(B) $2^x \log 2 + c$

(C) $2^x f(x) + c$

(D) $2^x + c$

Ans : (C)

Hint : $\int 2^x\left(f'(x) + f(x)\log 2\right)dx = I$

Let $g(x) = 2^x f(x)$

$\Rightarrow g'(x) = 2^x f'(x) + 2^x f(x)\log 2$

$= 2^x (f'(x) + f(x) \log 2)$

$\therefore I = \int g'(x)dx = g(x) + c = 2^x f(x)+c$

23. $\int_0^1 \log\left(\frac{1}{x}-1\right)dx =$

(A) 1 (B) 0 (C) 2 (D) None of these

Ans : (B)

Hint : Let $I = \int_0^1 \log\left(\frac{1}{x}-1\right)dx = \int_0^1 \log\left(\frac{1-x}{x}\right)dx$

$I = \int_0^1 \log\left(\frac{x}{1-x}\right)dx = -I$ $\left(\int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right)$

$\therefore 2I = 0 \Rightarrow I = 0$

24. The value of $\underset{n\to\infty}{\text{lt}}\left\{\frac{\sqrt{n+1}+\sqrt{n+2}+....+\sqrt{2n-1}}{n^{3/2}}\right\}$ is

(A) $\frac{2}{3}\left(2\sqrt{2}-1\right)$ (B) $\frac{2}{3}\left(\sqrt{2}-1\right)$

(C) $\frac{2}{3}\left(\sqrt{2}+1\right)$ (D) $\frac{2}{3}\left(2\sqrt{2}+1\right)$

Ans : (A)

Hint : $\underset{n\to\infty}{\text{lt}}\left(\frac{\sqrt{n+1}+\sqrt{n+2}+\sqrt{n+3}+......+\sqrt{2n-1}}{n^{3/2}}\right)$

$= \underset{n\to\infty}{\text{lt}}\left(\sqrt{1+\frac{1}{n}}+\sqrt{1+\frac{2}{n}}+.........+\sqrt{1+\frac{n-1}{n}}\right)\frac{1}{n}$

$= \underset{n\to\infty}{\text{lt}}\sum_{r=1}^{n-1}\frac{1}{n}\sqrt{1+\frac{r}{n}}$

$= \int_0^1\sqrt{1+x}\,dx = \frac{2}{3}.\left(2\sqrt{2}-1\right)$

25. If the solution of the differential equation $x\frac{dy}{dx}+y = xe^x$ be, $xy = e^x\varphi(x) + c$ then $\varphi(x)$ is equal to

(A) x+1 (B) x–1

(C) 1–x (D) x

Ans : (B)

Hint : If = $e^{\int\frac{dx}{x}} = e^{\ell n x} = x$

$\therefore xy = \int xe^x dx = (x-1)e^x + c$

26. The order of the differential equation of all parabolas whose axis of symmetry along x-axis is

(A) 2 (B) 3

(C) 1 (D) None of these

Ans : (A)

Hint : $y^2 = 4a(x-b)$

27. The line y = x + λ is tangent to the ellipse $2x^2 + 3y^2 = 1$. Then λ is

(A) –2 (B) 1

(C) $\sqrt{\frac{5}{6}}$ (D) $\sqrt{\frac{2}{3}}$

Ans : (C)

Hint : $\lambda^2 = \frac{1}{2} + \frac{1}{3} = 5/6 \Rightarrow \lambda = \sqrt{5/6}$

28. The area enclosed by y = $\sqrt{5-x^2}$ and y = |x–1| is

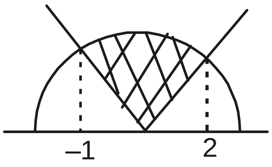
(A) $\left(\frac{5\pi}{4} - 2\right)$ sq. units (B) $\frac{5\pi - 2}{2}$ sq. units

(C) $\left(\frac{5\pi}{4} - \frac{1}{2}\right)$ sq. units (D) $\left(\frac{\pi}{2} - 5\right)$ sq. units

Ans : (C)

Hint : $\int_{-1}^{2} \sqrt{5-x^2}\, dx = 2 + \frac{5\pi}{4}$

$\int_{-1}^{2} |x-1|\, dx = \frac{5}{2}$



$\therefore$ Area = $\frac{5\pi}{4} - \frac{1}{2}$

29. Let S be the set of points whose abscissas and ordinates are natural numbers. Let P $\in$ S such that the sum of the distance of P from (8,0) and (0,12) is minimum among all elements in S. Then the number of such points P in S is

(A) 1 (B) 3

(C) 5 (D) 11

Ans : (B)

Hint : Sum of distances will be minimum if P, (8,0) and (0,12) will collinear

$\therefore \frac{x}{8} + \frac{y}{12} = 1 \Rightarrow y = 12 - \frac{3}{2}x$

$\therefore$ (x,y) $\equiv$ (2,9), (4,6), (6,3)

30. Time peirod T of a simple pendulum of length l is given by T = $2\pi\sqrt{\frac{l}{g}}$. If the length is increased by 2%, then an approximate change in the time period is

(A) 2% (B) 1%

(C) $\frac{1}{2}$% (D) None of these

Ans : (B)

Hint : $\dfrac{dT}{d\ell} = \dfrac{2\pi}{\sqrt{g}} \cdot \dfrac{1}{2\sqrt{\ell}}$

$\therefore\ \Delta T = \dfrac{dT}{d\ell}.\Delta\ell = \dfrac{\pi}{\sqrt{g\ell}} \cdot \left(\dfrac{2\ell}{100}\right)$

$= 2\pi\sqrt{\dfrac{\ell}{g}} \cdot \dfrac{1}{100} = \dfrac{T}{100}$

$\therefore\ \dfrac{\Delta T}{T} = \dfrac{1}{100}$

$\therefore$ 1%

31. The cosine of the angle between any two diagonals of a cube is

(A) $\dfrac{1}{3}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{2}{3}$ (D) $\dfrac{1}{\sqrt{3}}$

Ans : (A)

32. If x is a positive real number different from 1 such that $\log_a x$, $\log_b x$, $\log_c x$ are in A.P., then

(A) $b = \dfrac{a+c}{2}$ (B) $b = \sqrt{ac}$

(C) $c^2 = (ac)^{\log_a b}$ (D) None of (A), (B), (C) are correct

Ans : (C)

Hint : $2\log_b x = \log_a x + \log_c x = \dfrac{1}{\log_x a} + \dfrac{1}{\log_x c} \Rightarrow \dfrac{2}{\log_x b} = \dfrac{\log_x ac}{\log_x a \log_x c} \Rightarrow 2\log_x c = \dfrac{\log_x b}{\log_x a}(\log_x ac) = \log_a b.\log_x ac$

$\Rightarrow c^2 = (ac)^{\log_a b}$

33. If a, x are real numbers and $|a| < 1$, $|x| < 1$, then $1 + (1+a)x + (1+a+a^2)x^2 + ...\infty$ is equal to

(A) $\dfrac{1}{(1-a)(1-ax)}$ (B) $\dfrac{1}{(1-a)(1-x)}$ (C) $\dfrac{1}{(1-x)(1-ax)}$ (D) $\dfrac{1}{(1+ax)(1-a)}$

Ans : (C)

Hint : $\dfrac{1}{1-x} + \dfrac{ax}{1-x} + \dfrac{a^2x^2}{1-x} + ... = \dfrac{1}{1-x} \times \left(1 + ax + a^2x^2 + ...\right) = \dfrac{1}{1-x} \cdot \dfrac{1}{1-ax}$

34. if $\log_{0.3}(x-1) < \log_{0\ 09}(x-1)$, then x lies in the interval

(A) $(2, \infty)$ (B) $(1, 2)$ (C) $(-2, -1)$ (D) None of these

Ans : (A)

Hint : $\log_{0\ 3}(x-1) < \log_{(0.3)^2}(x-1) \Rightarrow \log_{0\ 3}(x-1)^2 < \log_{(0\ 3)}(x-1) \Rightarrow (x-1)^2 > x-1$ (0.3 < 1)

$\Rightarrow (x-1)(x-2) > 0 \Rightarrow x < 1, x > 2 \Rightarrow x > 2$ $(x \nless 1)$

35. The value of $\sum\limits_{n=1}^{13}\left(i^n + i^{n+1}\right), i = \sqrt{-1}$, is

(A) i (B) i–1 (C) 1 (D) 0

Ans : (B)

Hint : $\sum\limits_{n=1}^{13}\left(i^n + i^{n+1}\right) = i - 1$

36. If z_1, z_2, z_3 are imaginary numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1$ then $|z_1+z_2+z_3|$ is

(A) Equal to 1 (B) Less than 1 (C) Greater than 1 (D) Equal to 3

Ans : (A)

Hint : $z.\bar{z} = |z|^2 \rightleftharpoons \bar{z} = \frac{1}{z} \because \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right| = 1 \Rightarrow |\bar{z}_1+\bar{z}_2+\bar{z}_3| = 1 \Rightarrow |z_1+z_2+z_3| = 1$

37. If p, q are the roots of the equation $x^2 + px + q = 0$, then

(A) $p = 1, q = -2$ (B) $p = 0, q = 1$ (C) $p = -2, q = 0$ (D) $p = -2, q = 1$

Ans : (A)

Hint :

$$\begin{matrix} 2p^2 + q = 0 \\ q(p+q+1) = 0 \end{matrix} \Rightarrow p=1,\ q=-2$$

38. The number of values of k for which the equation $x^2 - 3x + k = 0$ has two distinct roots lying in the interval (0, 1) are

(A) Three (B) Two

(C) Infinitely many (D) No values of k satisfies the requirement

Ans : (C)

Hint : $f(0) > 0$ $f(1) > 0$ $\Rightarrow K > 2$ and $D > 0 \Rightarrow K < 9/4$ so $2 < K < 9/4$

39. The number of ways in which the letters of the word ARRANGE can be permuted such that the R's occur together is

(A) $\frac{\underline{|7}}{\underline{|2}\underline{|2}}$ (B) $\frac{\underline{|7}}{\underline{|2}}$ (C) $\frac{\underline{|6}}{\underline{|2}}$ (D) $\underline{|5} \times \underline{|2}$

Ans : (C)

Hint : A A $\boxed{RR}$ N G E. Number of arrangement = $\frac{\underline{|6}}{\underline{|2}}$

40. If, $\frac{1}{{}^5C_r} + \frac{1}{{}^6C_r} = \frac{1}{{}^4C_r}$, then the value of r equals to

(A) 4 (B) 2 (C) 5 (D) 3

Ans : (B)

41. For +ve integer n, $n^3 + 2n$ is always divisible by

(A) 3 (B) 7 (C) 5 (D) 6

Ans : (A)

42. In the expansion of $(x-1)(x-2) \ldots (x-18)$, the coefficient of x^{17} is

(A) 684 (B) –171 (C) 171 (D) –342

Ans : (B)

Hint : Coefficient of x^{17} is: $-(1+2+3+\ldots\ldots+18) = -\left(\frac{18\times19}{2}\right) = -171$

43. $1 + {}^nC_1 \cos\theta + {}^nC_2 \cos 2\theta + \ldots\ldots + {}^nC_n \cos n\theta$ equals

(A) $\left(2\cos\frac{\theta}{2}\right)^n \cos\frac{n\theta}{2}$ (B) $2\cos^2\frac{n\theta}{2}$ (C) $2\cos^{2n}\frac{\theta}{2}$ (D) $\left(2\cos^2\frac{\theta}{2}\right)^n$

Ans : (A)

Hint : Re $({}^{n}C_0 + {}^{n}C_1 e^{i\theta} + ...)$ = Re $(1 + e^{i\theta})^n$ = Re $(\cos\theta + 1 + i\sin\theta)^n$ $= \left(2\cos\left(\theta/2\right)\right)^n \cos\left(\frac{n\theta}{2}\right)$

44. If x, y and z be greater than 1, then the value of $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is

(A) log x. logy. log z (B) log x + logy + log z (C) 0 (D) $1-\{(\log x).(\log y).(\log z)\}$

Ans : (C)

Hint : $\begin{vmatrix} \frac{\log x}{\log x} & \frac{\log y}{\log x} & \frac{\log z}{\log x} \\ \frac{\log x}{\log y} & \frac{\log y}{\log y} & \frac{\log z}{\log y} \\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & \frac{\log z}{\log z} \end{vmatrix}$

Taking $\frac{1}{\log x}, \frac{1}{\log y}, \frac{1}{\log z}$ common from R_1, R_2, R_3 all rows are identical. So $\Delta=0$

45. Let A is a 3 × 3 matrix and B is its adjoint matrix. If |B| = 64, then |A| =

(A) ±2 (B) ±4 (C) ±8 (D) ±12

Ans : (C)

Hint : $|\text{Adj}(A)| = |A|^2 = 64 \Rightarrow |A| = \pm 8$

46. Let $Q = \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}$ and $x = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ then Q^3x is equal to

(A) $\binom{0}{1}$ (B) $\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ (C) $\binom{-1}{0}$ (D) $\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$

Ans : (C)

Hint : If $Q(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, $Q^3(\theta) = Q(3\theta)$, $Q^3(\pi/4) = \begin{bmatrix} \cos 3\pi/4 & -\sin 3\pi/4 \\ \sin 3\pi/4 & \cos 3\pi/4 \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$, $Q^3(\pi/4)x = \binom{-1}{0}$

47. Let R be a relation defined on the set Z of all integers and xRy when x + 2y is divisible by 3. Then

(A) R is not transitive (B) R is symmetric only

(C) R is an equivalence relation (D) R is not an equivalence relation

Ans : (D)

48. If $A = \{5^n - 4n - 1 : n \in N\}$ and $B = \{16(n-1) : n \in N\}$, then

(A) A = B (B) $A \cap B = \phi$ (C) $A \subseteq B$ (D) $B \subseteq A$

Ans : (C)

Hint : $5^n - 4n - 1 = (4+1)^n - 4n - 1 = 16k, k \in z$, A is a set of some multiple of 16 while set B is the set of all consecutive multiple of 16.

49. If the function $f : \mathbb{R} \to R$ is defined by $f(x) = (x^2+1)^{35}\ \forall \in \mathbb{R}$, then f is

(A) one-one but not onto (B) onto but not one-one

(C) neither one-one nor onto (D) both one-one and onto

Ans : (C)

Hint : $f(x) = (x^2+1)^{35}$

Since f(x) is even function hence not one one and f(x) > 0 $\forall x \in R$ hence not onto

50. Standard Deviation of n observations $a_1, a_2, a_3 .....a_n$ is σ. Then the standard deviation of the observations $\lambda a_1, \lambda a_2, ... \lambda a_n$ is

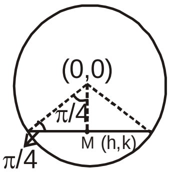
(A) $\lambda\sigma$ (B) $-\lambda\sigma$ (C) $|\lambda|\sigma$ (D) $\lambda^n\sigma$

Ans : (C)

CATEGORY - II (Q51 to Q65)

Only one answer is correct. Correct answer will fetch full marks 2. Incorrect answer or any combination of more than one answer will fetch – ½ marks.

51. The locus of the midpoints of chords of the circle $x^2+y^2=1$ which subtends a right angle at the origin is

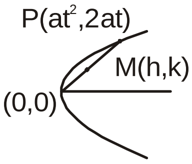


(A) $x^2+y^2=\frac{1}{4}$ (B) $x^2+y^2=\frac{1}{2}$ (C) xy = 0 (D) $x^2 - y^2 = 0$

Ans : (B)

Hint : $\sin\pi/4 = \frac{\sqrt{h^2+k^2}}{1}$, $h^2 + k^2 = 1/2$

52. The locus of the midpoints of all chords of the parabola $y^2 = 4ax$ through its vertex is another parabola with directrix



(A) x = – a (B) x = a (C) x = 0 (D) $x = -\frac{a}{2}$

Ans : (D)

Hint : $2h = at^2$, $2k = 2at \Rightarrow t = k/a, \Rightarrow 2h = a\frac{k^2}{a^2}$, $y^2 = 2ax$, Equation of its directri x = –a/2

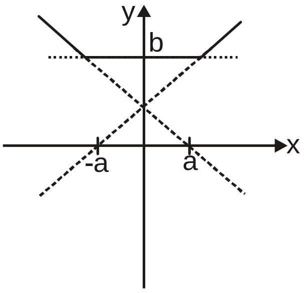
53. If [x] denotes the greatest integer less than or equal to x, then the value of the integral $\int_0^2 x^2[x]dx$ equals

(A) $\frac{5}{3}$ (B) $\frac{7}{3}$ (C) $\frac{8}{3}$ (D) $\frac{4}{3}$

Ans : (B)

Hint : $\int_0^2 x^2[x]\cdot dx = \int_0^1 x^2 \times 0dx + \int_1^2 x^2 \times 1dx = (x^3/3)_1^2 = 7/3$

54. The number of points at which the function f(x) = max {a – x, a + x, b}, $-\infty < x < \infty$, 0 < a < b cannot be differentiable



(A) 0 (B) 1 (C) 2 (D) 3

Ans : (C)

Hint : Possible graph of f(x) is as shown. There are to sharp turn, Hence f(x) cannot be differentiable at two point

55. For non-zero vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}+\vec{b}| < |\vec{a}-\vec{b}|$, then $\vec{a}$ and $\vec{b}$ are

(A) Collinear
(B) Perpendicular to each other
(C) Inclined at an acute angle
(D) Inclined at an obtuse angle

Ans : (D)

Hint : $|\vec{a}+\vec{b}| < |\vec{a}-\vec{b}| \Rightarrow |\vec{a}+\vec{b}|^2 < |\vec{a}-\vec{b}|^2$

$|\vec{a}|^2 + |\vec{b}| + 2|\vec{a}||\vec{b}|\cos\alpha < |\vec{a}|^2 + |\vec{b}| - 2|\vec{a}||\vec{b}|\cos\alpha$, (where α is an angle between $\vec{a}$ and $\vec{b}$ vector

$\Rightarrow 4|\vec{a}||\vec{b}|\cos\alpha < 0$, $\Rightarrow \cos\alpha < 0$, $\Rightarrow$ α is an obtuse angle

56. General solution of $y\frac{dy}{dx} + by^2 = a\cos x$, 0 < x < 1 is

(A) $y^2 = 2a(2b \sin x + \cos x) + ce^{-2bx}$
(B) $(4b^2 + 1)y^2 = 2a(\sin x + 2b\cos x) + ce^{-2bx}$
(C) $(4b^2 + 1)y^2 = 2a(\sin x + 2b\cos x) + ce^{2bx}$
(D) $y^2 = 2a(2b\sin x + \cos x) + ce^{-2bx}$
Here c is an arbitrary constant

Ans : (B)

Hint : Let $y^2 = z$

$$y\frac{dy}{dx} = \frac{1}{2}\frac{dz}{dx}$$

$$\frac{dz}{dx} + 2bz = 2a\cos x$$

$$IF = e^{2b\int dx} = e^{2bx}$$

$$z.e^{2bx} = \int 2a\cos x.e^{2bx}.dx$$

$$y^2e^{2bx} = \frac{2a}{4b^2+1}(\sin x + 2b\cos x)e^{2bx} + c$$

$$(4b^2 + 1)y^2 = 2a(\sin x + 2b\cos x) + ce^{-2bx}$$

57. The points of the ellipse $16x^2 + 9y^2 = 400$ at which the ordinate decreases at the same rate at which the abscissa increases is/are given by

(A) $\left(3, \frac{16}{3}\right) \& \left(-3, \frac{-16}{3}\right)$ (B) $\left(3, \frac{-16}{3}\right) \& \left(-3, \frac{16}{3}\right)$ (C) $\left(\frac{1}{16}, \frac{1}{9}\right) \& \left(-\frac{1}{16}, -\frac{1}{9}\right)$ (D) $\left(\frac{1}{16}, -\frac{1}{9}\right) \& \left(-\frac{1}{16}, \frac{1}{9}\right)$

Ans : (A)

Hint : $\frac{x^2}{25} + \frac{y^2}{\frac{400}{9}} = 1$

$(5\cos\theta, \frac{20}{3}\sin\theta)$

$x = 5\cos\theta$, $y = \frac{20}{3}\sin\theta$

$\frac{dx}{d\theta} = -5\sin\theta$, $\frac{dy}{d\theta} = \frac{20}{3}\cos\theta$

$\frac{dx}{d\theta} = -\frac{dy}{d\theta}$

$-5\sin\theta = -\frac{20}{3}\cos\theta$

$\tan\theta = 4/3$ $\Rightarrow \cos\theta = 3/5$ or $-3/5$

$\sin\theta = 4/5$ or $-4/5$

Points are $\left(3, \frac{16}{3}\right)$ and $\left(-3, \frac{-16}{3}\right)$

58. The letters of the word COCHIN are permuted and all permutation are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is

(A) 96 (B) 48 (C) 183 (D) 267

Ans : (A)

Hint : COCHIN

$$\underset{\text{4ways}}{C} + \underbrace{\_\ \_\ \_\ \_}_{\text{4/ways}} = 4 \times 4! = 96$$

59. If the matrix $A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{pmatrix}$, then $A^n = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ b & 0 & a \end{pmatrix}$, $n \in N$ where

(A) $a = 2n, b = 2^n$ (B) $a = 2^n, b = 2n$ (C) $a = 2^n, b = n2^{n-1}$ (D) $a = 2^n, b = n2^n$

Ans : (D)

Hint : $A = 2\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \Rightarrow A^n = 2^n\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ n & 0 & 1 \end{pmatrix}$

60. The sum of n terms of the following series; $1^3 + 3^3 + 5^3 + 7^3 + \ldots$ is

(A) $n^2(2n^2 - 1)$ (B) $n^3(n - 1)$ (C) $n^3 + 8n + 4$ (D) $2n^4 + 3n^2$

Ans : (A)

Hint : $t_r = (2r - 1)^3$

$$S_n = \sum_{r=1}^{n} t_r = 8\sum_{r=1}^{n} r^3 - 12\sum_{r=1}^{n} r^2 + 6\sum_{r=1}^{n} r - \sum_{r=1}^{n} 1 = n^2(2n^2 - 1)$$

61. If α and β are roots of $ax^2 + bx + c = 0$ then the equation whose roots are α^2 and β^2 is

(A) $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$

(B) $a^2x^2 + (b^2 - ac)x + c^2 = 0$

(C) $a^2x^2 + (b^2 + ac)x + c^2 = 0$

(D) $a^2x^2 + (b^2 + 2ac)x + c^2 = 0$

Ans : (A)

Hint : Let $y = x^2 \Rightarrow x = \sqrt{y}$

putting $\sqrt{y}$ in the given equation

$ay + b\sqrt{y} + c = 0 \quad \Rightarrow b\sqrt{y} = -ay - c \qquad \Rightarrow b^2y = a^2y^2 + c^2 + 2acy$

$\Rightarrow a^2y^2 - (b^2 - 2ac)y + c^2 = 0$

So the required quadratic equation is $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$

62. If ω is an imaginary cube root of unity, then the value of

$(2 - \omega)(2 - \omega^2) + 2(3 - \omega)(3 - \omega^2) + \ldots + (n - 1)(n - \omega)(n - \omega^2)$ is

(A) $\frac{n^2}{4}(n+1)^2 - n$ (B) $\frac{n^2}{4}(n+1)^2 + n$ (C) $\frac{n^2}{4}(n+1)^2$ (D) $\frac{n^2}{4}(n+1) - n$

Ans : (A)

Hint : $\sum_{r=2}^{n}(r-1)(r-\omega)(r-\omega^2) = \sum_{r=2}^{n}(r^3 - 1) = \left[\frac{n^2(n+1)^2}{4} - 1\right] - (n-1) = \frac{n^2(n+1)^2}{4} - n$

63. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$ then the value of ${}^nC_8$ is

(A) 10 (B) 7 (C) 9 (D) 8

Ans : (C)

Hint : $\frac{\underline{|n}}{\underline{|r-1}\,\underline{|n-r+1}} = 36$ ......(1)

$\frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}} = 84$ ....(2)

$\frac{\underline{|n}}{\underline{|r+1}\,\underline{|n-r-1}} = 126$ ......(3)

(1) ÷ (2) gives $\frac{r}{n-r+1} = \frac{36}{84} \quad \Rightarrow 84r = 36n - 36r + 36$ or $120r = 36n + 36$ ....(4)

(2) ÷ (3) gives $\frac{r+1}{n-r} = \frac{84}{126} \Rightarrow 126r + 126 = 84n - 84r$ or $210r = 84n - 126$ ....(5)

Solving (4) and (5) $n = 9, r = 3$

So ${}^nC_8 = {}^9C_8 = 9$

64. In a group 14 males and 6 females, 8 and 3 of the males and females respectively are aged above 40 years. The probability that a person selected at random from the group is aged above 40 years, given that the selected person is female, is

(A) $\frac{2}{7}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{5}{6}$

Ans : (B)

Hint : Here out of 6 females 3 are aged above 40 and 3 are aged below 40. So probability of person aged above 40 given female person = $\frac{1}{2}$

65. The equation $x^3 - yx^2 + x - y = 0$ represents

(A) a hyperbola and two straight lines

(B) a straight line

(C) a parabola and two straight lines

(D) a straight line and a circle

Ans : (B)

Hint : $x^3 - yx^2 + x - y = 0 \Rightarrow x^2(x - y) + (x - y) = 0$

$(x^2 + 1)(x - y) = 0$

So only possibility is $x = y$ as $x^2 + 1 \neq 0$

So it represents a straight line.

CATEGORY - III (Q66 to Q75)

One or more answer(s) is (are) correct. Correct answer(s) will fetch marks 2. Any combination containing one or more incorrect answer will fetch 0 marks. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked / actual number of correct answers.

66. If the first and the $(2n+1)^{th}$ t erms of an AP, GP and HP are equal and their n^{th} terms are respectively a, b, c then always

(A) $a = b = c$ (B) $a \geq b \geq c$

(C) $a + c = b$ (D) $ac - b^2 = 0$

Ans : (B, D)

Hint : There seems to be a printing mistake here

If there are (2n–1) terms instead of (2n + 1) terms then n^{th} terms of the A.P., G.P. and H.P. are the A.M., G.M. & H.M of the first and the last terms.

So, $a \geq b \geq c$ & $ac - b^2$ (B, D)

otherwise if there are (2n + 1) terms then the n^{th} terms should be in decreasing order of A.P., G.P. & H.P. i.e. $a \geq b \geq c$. (B)

67. The coordinates of a point on the line $x + y + 1 = 0$ which is at a distance $\frac{1}{5}$ unit from the line $3x + 4y + 2 = 0$ are

(A) (2, – 3) (B) (–3, 2)

(C) (0, –1) (D) (–1, 0)

Ans : (B, D)

Hint : Let $(t, -t - 1)$ be a parametric point of the line $x + y + 1 = 0$

Distance of $(t, -t - 1)$ from $3x + 4y + 2 = 0$ is

$$\frac{|3t+4(-t-1)+2|}{\sqrt{3^2+4^2}}=\frac{1}{5}$$

$\Rightarrow |-t-2| = 1 \Rightarrow |t+2| = 1$, so $t = -1$ or $t = -3$

possible co-ordinates are $(-1, 0)$ & $(-3, 2)$

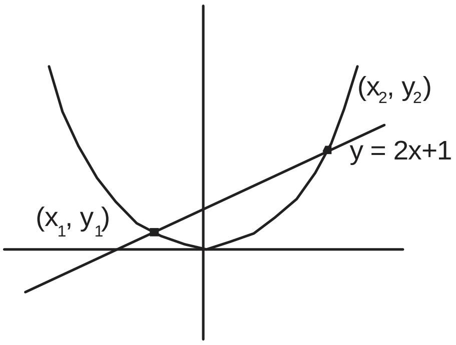
68. If the parabola $x^2 = ay$ makes an intercept of length $\sqrt{40}$ unit on the line $y - 2x = 1$ then a is equal to

(A) 1 (B) – 2

(C) –1 (D) 2

Ans : (A, B)

Hint :



Solving $x^2 = ay$ with $y - 2x = 1$,

$x^2 = a(1 + 2x) \Rightarrow x^2 - 2ax - a = 0$

Let x_1 & x_2 are the roots

so, $(x_1 - x_2)^2 = (2a)^2 - 4(-a) = 4a(a+1)$

also, $(y_1 - y_2)^2 = ((2x_1+1)-(2x_2+1))^2 = 4(x_1 - x_2)^2 = 16a(a+1)$

now $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2} = \sqrt{4a(a+1)+16a(a+1)} = \sqrt{40}$

$\Rightarrow 20a(a + 1) = 40 \Rightarrow a^2 + a - 2 = 0 \Rightarrow a = -2, 1$

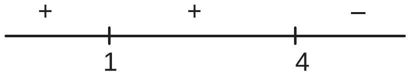
69. if f(x) is a function such that $f'(x) = (x-1)^2(4-x)$, then

(A) $f(0) = 0$ (B) f(x) is increasing in (0, 3)

(C) x = 4 is a critical point of f(x) (D) f(x) is decreasing in (3, 5)

Ans : (B, C)

Hint : $f'(x) = (x-1)^2(4-x)$



The sign scheme of $f'(x)$

so clearly f(x) is increasing in (0, 3) as $f'(x) \geq 0$. (B)

x = 4 is a critical point as $f'(4) = 0$. (C)

from $f'(x)$, we can't determine f(x) uniquely so f(0) can't be predicted

70. On the ellipse $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are

(A) $\left(\frac{2}{5}, \frac{1}{5}\right)$

(B) $\left(-\frac{2}{5}, \frac{1}{5}\right)$

(C) $\left(-\frac{2}{5}, -\frac{1}{5}\right)$

(D) $\left(\frac{2}{5}, -\frac{1}{5}\right)$

Ans : (B, D)

Hint : Let $\left(\frac{1}{2}\cos\theta, \frac{1}{3}\sin\theta\right)$ be a point on $4x^2 + 9y^2 = 1$, so equation of tangent at $\left(\frac{1}{2}\cos\theta, \frac{1}{3}\sin\theta\right)$ is

$2x \cos\theta + 3y \sin\theta = 1$

equating slope with $8x = 9y$

$$\frac{-2\cos\theta}{3\sin\theta} = \frac{8}{9} \Rightarrow \tan\theta = -\frac{3}{4}$$

Hence either $\cos\theta = -\frac{4}{5},\ \sin\theta = \frac{3}{5}$

or $\cos\theta = \frac{4}{5},\ \sin\theta = -\frac{3}{5}$

so the points are $\left(-\frac{2}{5}, \frac{1}{5}\right)$ or $\left(\frac{2}{5}, -\frac{1}{5}\right)$

71. If $\varphi(t) = \begin{cases} 1, & \text{for } 0 \le t < 1 \\ 0 & \text{otherwise} \end{cases}$ then $\int_{-3000}^{3000} \left(\sum_{r'=2014}^{2016} \varphi(t - r')\varphi(t - 2016) \right) dt =$

(A) a real number

(B) 1

(C) 0

(D) does not exist

Ans : (A, B)

Hint : $\int_{-3000}^{3000} \varphi(t-2016)(\varphi(t-2014) + \varphi(t-2015) + \varphi(t-2016)).dt$

$$\int_{-3000}^{2016} 0.dt + \int_{2016}^{2017} 1.(0+0+1).dt + \int_{2017}^{3000} 0.dt = 1$$

72. If the equation $x^2 + y^2 - 10x + 21 = 0$ has real roots $x = a$ and $y = \beta$ then

(A) $3 \le x \le 7$

(B) $3 \le y \le 7$

(C) $-2 \le y \le 2$

(D) $-2 \le x \le 2$

Ans : (A, C)

Hint : $x^2 - 10x + (y^2 + 21) = 0$

for real roots of x, $D \ge 0$

$100 - 4(y^2 + 21) \ge 0$

$\Rightarrow y^2 \le 4$

$\Rightarrow -2 \le y \le 2$ (C)

also, $y^2 = -x^2 + 10x - 21$

for real roots of y,

$-x^2 + 10x - 21 \geq 0$

$\Rightarrow (x-7)(x-3) \leq 0$

$3 \leq x \leq 7$ (A)

73. If $z = \sin\theta - i\cos\theta$ then for any integer n,

(A) $z^n + \frac{1}{z^n} = 2\cos\left(\frac{n\pi}{2} - n\theta\right)$

(B) $z^n + \frac{1}{z^n} = 2\sin\left(\frac{n\pi}{2} - n\theta\right)$

(C) $z^n - \frac{1}{z^n} = 2i\sin\left(n\theta - \frac{n\pi}{2}\right)$

(D) $z^n - \frac{1}{z^n} = 2i\cos\left(\frac{n\pi}{2} - n\theta\right)$

Ans : (A, C)

Hint : $z = \sin\theta - i\cos\theta$

$$\cos\left(\theta - \frac{\pi}{2}\right) + i\sin\left(\theta - \frac{\pi}{2}\right)$$

$$= e^{i(\theta - \frac{\pi}{2})}$$

so, $z^n = e^{i\left(n\theta - \frac{n\pi}{2}\right)} = \cos\left(n\theta - \frac{n\pi}{2}\right) - i\sin\left(n\theta - \frac{n\pi}{2}\right)$

$\frac{1}{z^n} = e^{i\left(\frac{n\pi}{2} - n\theta\right)} = \cos\left(n\theta - \frac{n\pi}{2}\right) - i\sin\left(n\theta - \frac{n\pi}{2}\right)$, so $z^n + \frac{1}{z^n} = 2\cos\left(n\theta - \frac{n\pi}{2}\right) = 2\cos\left(\frac{n\pi}{2} - n\theta\right)$ (A)

$z^n - \frac{1}{z^n} = 2i\sin\left(n\theta - \frac{n\pi}{2}\right)$ (C)

74. Let $f : X \to X$ be such that $f(f(x)) = x$ for all $x \in X$ and $X \subseteq R$, then

(A) f is one-to-one

(B) f is onto

(C) f is one-to-one but not onto

(D) f is onto but not one-to-one

Ans : (A, B)

Hint : $f(f(x)) = x.\ \forall x \in X$

so, $f(x) = f^{-1}(x)$ i.e.f(x) is self invertible

Hence f(x) has to be one-one & onto

75. If A, B are two events such that $P(A \cup B) \geq \frac{3}{4}$ and $\frac{1}{8} \leq P(A \cap B) \leq \frac{3}{8}$ then

(A) $P(A) + P(B) \leq \frac{11}{8}$

(B) $P(A).P(B) \leq \frac{3}{8}$

(C) $P(A) + P(B) \geq \frac{7}{8}$

(D) None of these

Ans : (A, C)

Hint : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A) + P(B) = P(A \cup B) + P(A \cap B)$

$\frac{3}{4} \le P(A \cup B) \le 1$

$\frac{1}{8} \le P(A \cap B) \le \frac{3}{8}$

so, $\frac{7}{8} \le P(A \cup B) + P(A \cap B) \le \frac{11}{8}$

so, $P(A) + P(B) \ge \frac{7}{8}$ (C)

$P(A) + P(B) \le \frac{11}{8}$ (A)

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